A Multi-Objective Covering Tour Routing Model for Emergency Relief Logistics

Mathematical Model and Solution Methodology

Case study: earthquake response, Tehran Relief & Rescue Organization

1 Overview

This document presents the optimization model and solution methodology developed for an emergency relief-logistics problem: after a major earthquake, scarce relief vehicles must serve a very large set of population points in the shortest possible time, while prioritizing the most populous and most critical locations and respecting the limited capacity and range of the fleet.

The problem combines three classical streams of supply-chain research that are usually treated separately—**maximal covering**, **clustering (facility location)**, and **vehicle routing**—into a single integrated *covering tour routing problem*. Two real-world features distinguish it. First, not every demand point must be visited: a point may instead be *covered* by a nearby visited point, which is essential when there are tens of thousands of points and time is critical. Second, the road network is unreliable, because an earthquake damages roads and activates fault lines; the model therefore weights travel by the *probability that each road segment remains usable.*

Because solving the full problem at real scale (on the order of 10^4 points) is computationally intractable, the work adopts a **three-stage solution architecture**: (i) reduce the point set by maximal coverage; (ii) partition the survivors into clusters and locate one relief base at each cluster center; and (iii) solve a vehicle routing problem within each cluster using the Biogeography-Based Optimization (BBO) metaheuristic, validated against an exact solver on small instances.

2 Problem Definition and Three-Stage Architecture

The case study targets the Tehran metropolitan area, where a network of population points must be served from relief depots following an earthquake. Three considerations make a direct, single-shot routing model impractical: the sheer number of points; the splitting of the city into disconnected sub-regions when fault lines activate and roads collapse; and the limited fleet and resources of the relief organization. The solution therefore proceeds in three stages.

2.1 Stage 1 — Maximal Coverage (point reduction)

Each population point carries (a) a *population* and (b) a *weight* reflecting its geophysical and locational desirability, scored on a signed scale by the relief organization.

Positive factors (raise the weight): points designated by the organization as mandatory visits (+100), proximity to highways, to unroofed public open spaces, to large green spaces, to hospitals, to bus terminals, and to airports, and being on flat (plain) terrain.

Negative factors (lower the weight): lying within a 5 km radius of a fault line (−100), mountainous or foothill terrain, weak soil, and sitting over groundwater.

A coverage radius is assigned to each point in inverse proportion to its population,

$$r_i = \frac{\sum_\ell \mathrm{pop}_\ell}{\mathrm{pop}_i}\,\alpha, \tag{1}$$

so that more populous points receive a *smaller* radius (they must be visited directly rather than absorbed into a neighbor). The coverage procedure then operates as follows:

1. Select a point at random. Suppose the radii of K other points intersect its radius, giving a candidate group of $K+1$ points.
2. The point of *highest weight* in the group becomes the reference (visited) point; the others are covered by it. Points are assigned starting from the lowest weight and continuing until the per-point population limit would be violated.
3. Ties in weight are broken by population: a larger-population point covers a smaller-population one.
4. **Rule 1.** If the random point has negative weight and all intersecting points are also negative, it is re-selected; but if a positive-weight point is present, that positive point covers the random point.
5. **Rule 2.** If the random point and all intersecting points exceed $+100$, none is covered—these are organization-mandated must-visit points and may never be removed.
6. Any point that has covered others, or has itself been covered, is excluded from subsequent selection.

The output is a substantially reduced set of representative (to-be-visited) points, each carrying the aggregated population of the points it covers.

2.2 Stage 2 — Clustering and Base Location

Because activated faults effectively partition the city into sub-regions between which travel is difficult or impossible, the surviving points are grouped by **K-means clustering**. The optimal number of clusters is chosen by minimizing the **Davies–Bouldin index**, which favors clusters that are internally similar and well separated from one another. A relief base (depot) is then located at the center of each cluster, and routing is performed independently within each cluster.

2.3 Stage 3 — Vehicle Routing

Within each cluster a capacitated, distance-limited vehicle routing problem with simultaneous pickup (of injured people) and delivery (of relief goods) is solved. The objective minimizes the *reliability-weighted, population-prioritized* travel distance. The full model is given next.

3 Routing Model: Notation

Symbol	Definition
Sets and indices	

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N	Set of all population points plus the depot, $N = \{0, 1, \ldots, n\}$; node 0 is the hub (depot)
K	Set of homogeneous vehicles, $K = \{1, \ldots, k\}$
i, j	Point indices, $i, j \in N$
k	Vehicle index, $k \in K$
Parameters	
C_{ij}	Distance from point i to point j (km)
U_i	Population of point i (including the population of all points covered by i)
W_{ij}	Edge priority weight: 0.4 if $U_i < U_j$ and 0.6 if $U_i > U_j$
R_{ij}	Probability that the link $i \to j$ remains usable, $R_{ij} = e^{-\gamma C_{ij}}$
E_{ij}	Penalty / detour-inflation factor applied when the link is *not* usable
d_i	Number of demand units to be delivered at point i
p_i	Number of units (injured) to be picked up at point i
Q	Capacity of each vehicle
L	Maximum route length a vehicle may travel
n	Total number of points, $n = \|N\|$
γ	Decay coefficient of link reliability ($\gamma = 0.00009$)
Decision variables	
x_{ijk}	Binary; $= 1$ if vehicle k traverses link $i \to j$, else 0
y_{ij}	Integer; units *picked up* so far and carried on link (i, j)
z_{ij}	Integer; units *to be delivered* carried on link (i, j)

Note on E_{ij} and W_{ij}. The factor E_{ij} is left only partially specified in the source; it is interpreted here as the cost inflation incurred when a primary link is unusable and a detour must be taken. The asymmetric weights W_{ij} encode the population-priority rule: moving toward a more populous point is rewarded relative to moving away from it.

4 Routing Model: Formulation

The proposed routing problem is an integer linear program.

4.1 Objective

$$\min \sum_{i=0}^{N} \sum_{j=0}^{N} \sum_{k=1}^{K} \Big[R_{ij}\, W_{ij}\, C_{ij} \; + \; \big(1 - R_{ij}\big)\, W_{ij}\, C_{ij}\, E_{ij} \Big]\, x_{ijk} \tag{2}$$

The objective minimizes the *expected* weighted travel distance of the relief fleet. For each traversed link, with probability R_{ij} the road is usable and the incurred cost is $W_{ij}C_{ij}$; with probability $1 - R_{ij}$ the road is damaged and an inflated detour cost $W_{ij}C_{ij}E_{ij}$ applies. The edge weights W_{ij} simultaneously bias route construction toward higher-population points.

4.2 Constraints

$$\sum_{i=0}^{N}\sum_{k=1}^{K} x_{ijk} = 1, \qquad \forall\, j = 1,\dots,N, \tag{3}$$

$$\sum_{i=0}^{N} x_{ijk} = \sum_{i=0}^{N} x_{jik}, \qquad \forall\, j = 0,\dots,N,\ \forall\, k = 1,\dots,K, \tag{4}$$

$$\sum_{j=1}^{N} x_{0jk} \le 1, \qquad \forall\, k = 1,\dots,K, \tag{5}$$

$$\sum_{i=1}^{N} x_{i0k} \le 1, \qquad \forall\, k = 1,\dots,K, \tag{6}$$

$$\sum_{i=0}^{N} y_{ji} - \sum_{i=0}^{N} y_{ij} = p_j, \qquad \forall\, j = 1,\dots,N, \tag{7}$$

$$\sum_{i=0}^{N} z_{ij} - \sum_{i=0}^{N} z_{ji} = d_j, \qquad \forall\, j = 1,\dots,N, \tag{8}$$

$$y_{ij} + z_{ij} \le Q \sum_{k=1}^{K} x_{ijk}, \qquad \forall\, i, j = 0,\dots,N, \tag{9}$$

$$\sum_{i=0}^{N}\sum_{j=0}^{N} C_{ij}\, x_{ijk} \le L, \qquad \forall\, k = 1,\dots,K, \tag{10}$$

$$\sum_{i=0}^{N}\sum_{j=0}^{N} x_{ijk} \le n - 1, \qquad \forall\, k = 1,\dots,K. \tag{11}$$

The constraints read as follows. Constraint (3) requires each demand point to be served exactly once, by exactly one vehicle. Constraint (4) is flow conservation: a vehicle that enters a node must leave it, guaranteeing route continuity. Constraints (5) and (6) limit each vehicle to at most one departure from and one return to the depot. Constraints (7) and (8) are the pickup and delivery balances, accumulating the picked-up (p_j) and to-be-delivered (d_j) loads along the route. Constraint (9) ensures the combined load (picked up plus to-deliver) carried on any link never exceeds the vehicle capacity Q. Constraint (10) caps each route length at the maximum range L. Constraint (11) is a subtour-elimination condition.

5 Solution Methodology: Biogeography-Based Optimization

Vehicle routing problems are NP-hard, so exact solvers are impractical at large scale. The routing stage is therefore solved with **Biogeography-Based Optimization (BBO)**, an evolutionary metaheuristic introduced by Simon (2008) and inspired by the migration of species between habitats (islands). BBO is attractive here because it aims to combine the solution *accuracy* characteristic of genetic algorithms with the convergence *speed* of particle-swarm optimization.

5.1 Biogeographic Analogy

A candidate solution is a *habitat*, described by a vector of suitability index variables (SIVs):

$$\text{Habitat} = [\,\text{SIV}_1, \text{SIV}_2, \ldots, \text{SIV}_n\,]. \tag{12}$$

The quality of a habitat is its Habitat Suitability Index (HSI), i.e. its fitness,

$$\text{HSI} = f(\text{Habitat}) = f(\text{SIV}_1, \ldots, \text{SIV}_n). \tag{13}$$

High-HSI habitats host many species; they consequently have a *high emigration* rate and a *low immigration* rate, and their species distribution is comparatively stable. Optimization proceeds by letting good solutions share their features (emigrate) with weaker solutions (immigrate), interleaved with occasional mutation.

5.2 Migration and Mutation Rates

For a habitat holding k species out of a maximum of n, the immigration rate λ_k and emigration rate μ_k are

$$\lambda_k = I\left(1 - \frac{k}{n}\right), \qquad \mu_k = E\,\frac{k}{n}, \tag{14}$$

where I and E are the maximum immigration and emigration rates (taken equal, $E = I$). Mutation perturbs an SIV with a probability that depends on the species-count probability P_s,

$$m_s = m_{\max}\left(1 - \frac{P_s}{P_{\max}}\right). \tag{15}$$

5.3 Generic BBO Steps

1. **Initialize parameters**: maximum number of species $S_{\max}$, maximum migration rate $E = I$, maximum mutation rate $m_{\max}$, and the elitism count.
2. **Generate the initial population** of habitats at random.
3. **Evaluate HSI** (fitness) for every habitat.
4. **Sort** the population by fitness and retain the elite solutions.
5. **Compute** the immigration (λ) and emigration (μ) rates.
6. **Migration**: modify each habitat by importing SIVs from others, with probabilities governed by λ and μ.
7. **Mutation**: randomly alter SIVs according to m_s.
8. Repeat the migration–mutation cycle until the stopping criterion is met, returning the best habitat found.

5.4 BBO Tailored to the Routing Problem

Solution encoding. An initial habitat is built by selecting p subtours at random from the n network points. Each row of the solution matrix represents one subtour: its first cell holds the subtour's center point and the remaining cells hold randomly chosen points. Each point may appear in only one subtour.

Fitness. Each habitat is scored using objective (2). Because the problem is a minimization, a normalized selection index is used,

$$\mu = \frac{1/f_i}{1/f_{\text{best}}} \in [0,1], \tag{16}$$

where values closer to 1 indicate fitter solutions. The n habitats with the highest μ are retained as elites and a further n_2 are passed to the mutation stage.

Mutation operators. Five neighborhood moves are used to diversify the population:

M1. Select two subtours; swap one randomly chosen point from each.

M2. Select two subtours; move a point from one subtour into the other.

M3. Select one subtour; detach a point to form a new subtour of its own.

M4. Select one subtour; detach two points to form a new subtour.

M5. Select one subtour; swap two of its points with each other.

After each move the fitness of the new population is recomputed, and the migration–mutation cycle continues until convergence.

6 Computational Study

6.1 Data and Tool Chain

The case study draws on roughly 11,840 population points across Tehran. The workflow chains several tools, each handling the stage it is best suited to:

Stage / quantity	**Tool and method**
Point coordinates & population	Acquired from Google Earth, with geophysical and surface attributes (faults, terrain type, proximity to highways, hospitals, open spaces, etc.)
Inter-point distances	Computed in ArcGIS, accounting for slope and elevation differences
Maximal coverage	Implemented in MATLAB (population cap ≈ 15,000, $\alpha = 20$, $\gamma = 0.00009$)
Clustering / base location	K-means in RapidMiner, optimal k via the Davies–Bouldin index
Routing (per cluster)	Biogeography-Based Optimization, coded in MATLAB
Validation (small instances)	Exact solution in GAMS

6.2 Procedure

After coverage reduces the point set and clustering partitions it, the population points of each cluster are routed independently through the BBO solver. On small instances the BBO solutions are checked against the exact optimum obtained in GAMS to confirm validity before the method is applied to the full-scale clusters.

7 Summary

The problem is a multi-objective covering tour routing problem for post-earthquake relief that, unusually, fuses maximal coverage, clustering-based facility location, and capacitated vehicle routing into one integrated workflow, with two realism-driven features: demand points may be covered rather than visited, and travel cost is weighted by the probability that each road remains usable after the disaster. Because the real instance is far too large for exact methods, a three-stage architecture is used—coverage-based point reduction, K-means clustering with Davies–Bouldin selection to site relief bases, and a Biogeography-Based Optimization metaheuristic to route vehicles within each cluster, with population-priority edge weights and simultaneous pickup and delivery. The routing model is an integer linear program minimizing expected reliability-weighted distance subject to single-visit, flow-conservation, fleet-size, pickup/delivery-balance, capacity, route-length, and subtour-elimination constraints, and the BBO solver is validated against GAMS on small instances before being deployed on the Tehran case study.