# Solving Multi-objective location-allocation problem under congestion in the uncertainty atmosphere of the emergence of customers with environmental consideration


## Abstract

Today, in most industrial and service centers, the problem of location-allocation of facilities, determining the location of facilities and assigning customers to them have been the main problems. In this research, a problem of location-allocation prone congestion is considered with the possibility of withdrawal of customers before arrival. On the other hand, due to the importance of environmental issues in the supply chain, the cost of pollution caused by the flow of flow between a customer and a service center as a green criterion is considered. In the nonlinear model presented in this study, three objectives have been investigated. The first goal is to minimize the maximum queuing available on the network, the second goal is to minimize the cost of pollution caused by the flow of flows between a customer and a service center, and the third goal is to minimize the number of customers to withdraw. Depending on the specific conditions in Np-hard issues, evolutionary algorithms are used to solve them. In this research, two solving phases have been used. In the first phase, the solving of fitness functions defined in the data space of CAB and AP is solved using the genetic algorithm and their results are analyzed. In the second phase of the solution, it is assumed that the time of the appearance of customer demand is randomly and follow a specific distribution. In this phase, the mathematical model is designed by a continuous locating problem with limited capacity, which on the other hand, according to some hypotheses, offers a discrete approximation model and allows for the allocation of facilitation in the candidate points. Therefore, it is hypothetically regionalized in two parts of the north and south, where the response rate in the facility is different in the north with the southern region. Finally, the proposed model is solved with numerical examples. In this regard, initially, in the proposed model, assumptions such as statistical population distribution, the statistical distribution of facilities and statistical distribution of customers' input have been determined. According to these, the problem is solved in three scenarios using genetic algorithm and Monte Carlo simulation. Finally, their results are compared and analyzed with charts and figures.



**Keyword:** uncertainty, location-allocation, green supply chain, genetic algorithm, Monte Carlo simulation.

## 1. Introduction

Mathematical models of MCLP (Maximal Covering Location Problem) can be different, depending on the space, where the problem is defined. Three different spaces, as discrete, continuous, and network spaces have so far been considered for those types of problems. It is assumed that the demand points and the set of candidate centers in providing services in the discrete space are definite points of the space. The distance between each demanding point and each service center is specified in such problems, and if the distance is not exceeding the default distance, there will be the possibility for providing the service of the demand point by the service center. Otherwise, the service of the demand point will be rejected by the center. The assumption is applied to the problem of increasing the service quality and the possibility of customers' services at the appropriate time. Although MCLP could show appropriate functions in some problems, it has not been successful in some others (Erkut, Ingolfsson, Sim, & Erdoğan, 2009). This problem can specifically be seen in compact and crowded systems. Various generalities have so far been given for eliminating the MCLP. The definite generalizations provided for the MCLP problem are useful in some cases. But despite problems with practical issues, researchers have been urging researchers to consider more realistic concepts in their research issues. One of these concepts was the use of queuing concepts. In real-world issues, because there are limitations on the number of service providers, each server at all times will often be able to respond to an applicant. Therefore, other customers will have to wait in order to receive the service for a while. Reducing the average waiting time in the queue can increase the quality of service and increase customer satisfaction

This remain of this paper is organized as follows; in Section 2 the literature review is presented, followed by the problem statement in Section 3. Section 4 characterizes in detail the mathematical model developed and in Section 5 solution approaches are proposed and explored in detail. Followed by algorithm results analysis and discussion in Section 6. To finalize Section 7 presents the conclusion and some final remarks on future work.

## 2. Literature review

Larson (1974, 1975) is the first researcher who had investigated the congestion phenomenon in the positioning of the facilities. Positioning models for the facilities susceptible to congestion are categorized according to the criteria such as the service providers in each facility, the basis of allocating the customers to the facilities (including proximity and gravity rules), distributing the times between successive entrances of the customers, and distributing the service providing times. The most important distinctive basis of these models is the type of fitness function, and in this respect, they can be categorized into three groups for the median, coverage, and central matters. The fitness function of the median models is defined as minimizing the total costs or times (including the costs or the traveling and waiting costs for the customers). The model of a service provider M/M/1(the Poisson inputs and the exponential service providing times) and some service providers M/M/c that are presented by Wang, Batta, & Rump (2002) and Berman & Drezner (2007), respectively, are two examples of median models. Using multi-purpose models is also considered by the researchers in this field of studies. For instance, in the mathematical models provided by Pasandideh & Niaki (2012) and Chambari, Rahmaty, Hajipour, & Karimi (2011), both aspects of the customer and the service provider are considered simultaneously within the framework of two separate functions. In contrast to the median models, the concentration of the fitness function in the covering models is on the quantity of the coverages instead of the quality of the customers' coverages. Thus, the fitness functions such as maximizing the rate of coverage or minimizing the customers out or the limits of the facilities coverage are used. The first model for the coverage problem by using queuing systems is related to the years 1994 and 1996 (Marianov & Revelle, 1994, 1996). Marianov and Serra (1998) considered an MCLP case by adding limitations for the average waiting time in the queue for the customers. They analyzed the MCLP with two queuing systems as M/M/I and M/M/m (or the number of fixed service providers for each service center) .Maximizing the expectancy of the number of covered customers (Berman, Huang, Kim, & Menezes, 2007) and maximizing the covering area (Hamaguchi & Nakade, 2010) and (M. Moghadas & Kakhki, 2011) can be named as such fitness functions. The fitness function of central positioning models, as the third positioning cases of the highly crowded facilities, considers minimizing the max. Distance or interval between the services. Application of this type of fitness function is exclusive to the positioning of the emergency facilities; i.e. the cases that require the shortest time to the farthest customer for giving the necessary service. The model provided by Aboolian, Berman, & Drezner (2009) can be pointed as an example for the central positioning models of highly crowded facilities. On the other hand, some of the customers in the real world service providing cases withdraw from receiving the service due to long queues before entering in the queuing system or after going into the queue. If the ration of such customers to the total customers is considerable, ignoring it can lead to uncertain results. Although various mathematical models are developed to analyze the impatience of the customers, such models have often been considered as the basic models in the queuing theory, and the applications such as positioning the congestive facilities are not properly considered. The reason for it can be in difficulty in applying balancing equations of the queuing system in the related mathematical model. Withdrawal from receiving the service due to impatience has occurred in three different forms; withdrawal before entry (balking) is the first form of impatience that occurs due to the reluctance of the customer to enter long queues. The queuing model M/M/C/K (Poisson entries; exponential service times, “C” service providers in each facility, and the waiting space limit for “k” number of people) is among the queuing models that involve this type of impatience. The presented model by Chambray et al. (2011) is a specific state of such a system, in which it is assumed that only one service provider is deployed in each facility (M/M/1/K). It is assumed in such a system that the customer avoids entering the queue by observing “k” number of people in the system. A more realistic model is given by Berman et al. (2007); i.e. it is assumed that the customer withdraws from the system by observing “k” people and goes to another facility. Although the conditions of the real world are considered to a great extent in this research, the authors have not applied the balancing equations of the queuing system in the mathematical model, and only regarded approximate methods. In a more realistic way, the reluctance of the customers to enter the queue depends on the number of present people in the queue; i.e. each customer decides about entering in or withdrawing from the queue with regards to the present people in the queue, by the time of going to his corresponding facility. By increasing the queue length, the rate of entering into each facility is reduced, which is considered within the frame of a reducing function appropriate with the existing people in each facility. The second form of impatience id withdrawal after entry (reneging). In this type of impatience, the customer estimates his waiting time with regards to the people ahead of him. If the estimated time by the customer is more than his expected time, he will not wait and leaves the facility before receiving the required service. Insisting (jockeying) is the third type of impatience, by which the customer does not wait in a queue for receiving the service and leaves the queue by seeing the reduction in the other queues to join to a shorter queue. In his article, A.M.K.Tarabia analyzes two parallel queues with limited capacities with the jockeying state to receive the required service. It is to note that the considered queuing systems in this study are M/M/1 and M/M/2. (Tarabia, 2008)

## 2.1. Methods of solving optimization problems

An algorithm is a collection of steps that are appropriate to the pursuit of an optimal point. Designing an algorithm can have several goals, including achieving a local optimal point, achieving an optimal global point, finding all the optimal global points, finding all of the global and local optimal points (Hendrix & Toth, 2010). In general, the algorithms presented in the field of optimization of the compounds are categorized into two groups of complete algorithms and approximate algorithms. Complete algorithms that involve finding an optimal solution in finite time are all finite (limited) optimization problems for all examples (Blum & Roli, 2008). Approximate algorithms were officially introduced in the 1960s to produce near-optimal solutions. These algorithms are used for optimization problems that cannot be effectively solved by computational methods at low rates. With the advent of Tom's non-Thesis theory in the early 1970s, this field became more important as the need to produce near-optimal answers to some very difficult optimization problems was strongly felt in order to overcome the complexity of these issues. The algorithms of that decade calculated the near-optimal answer in a short time for a problem and, for other issues, also hardly produced sub-optimal answers (Gonzales, 2007). Approximate methods are generally based on two basic principles: (1) constructive engineering techniques, and (2) local search methods. The innovative engineering techniques relate to the process of making the initial answers before performing other operations. Local search can be considered as an innovative mechanism in which neighbors of the current answer are considered as potential alternatives. If one of the neighbors of the current answer is accepted, the move to the new answer begins and the neighbors of the answer are taken into consideration (Burke and Kendall, 2005). Their innovative techniques are divided into two families of special initiatives and extraversions (Gonzales, 2007). Special initiatives are designed to solve a particular problem or an instance of it, while meta-initiatives are general-purpose algorithms that can be used in almost all optimization problems.

## 2.2. Meta heuristic Algorithm

In recent decades, a new type of approximate algorithm has emerged, which essentially aims to combine innovative methods in the framework of the cloud to explore the efficient and effective search space. Today, these methods are referred to as Meta heuristic methods. In recent decades, a new type of approximation algorithm has emerged, which essentially aims to combine innovative methods in larger frameworks to explore the efficient and effective search space. Today, these methods are referred to as meta-curiosity methods. This term was first used by Glover in 1986, which is made up of a combination of the two Greek words Meta and heuristic. Meta prefix means more or higher, and heuristic means to find. Before the general acceptance of the term fragmentation, the term "innovative method" was used for such methods (Blum & Roli, 2003). A meta-modern method is, in fact, an innovative method for solving a very general class of issues, and is an efficient combination of objective functions or innovative methods based on objective functions (Weise, 2007). The main advantage of using ultra-innovative methods is the limited assumptions in formulating the model, while this does not apply to mathematical programming. Some of the optimization problems cannot be formulated with unambiguous analytical-mathematical symbols. In fact, the objective function in these cases is like a black box. In optimizing the black box, there are no analytical rules for the purpose (Blum & Roli, 2003). So far, many meta-innovative algorithms have been proposed. These algorithms have higher performance for some issues, and in some cases they have problems with approaching the optimal answer. Many of the major meta-bureaucracies are of natural or physical processes. Anecdotal Optimization (ACO), evolutionary algorithms (EAs) such as Genetic Algorithm (GA) and Simulated Annealing (SA) are examples of such algorithms. ACO and EAs of nature and SA are inspired by the process of cooled metals in the production of solid metals (Borenstein & Poli, 2008).

One of the most efficient meta-algorithms in solving complex optimization problems is the complexity of the genetic algorithm inspired by the theory of natural evolution. GA considers the answer to the concept of chromosomes that is composed of genes. Chromosomes are combined with mutant and intersecting operators, and create the answers of the child and, consequently, the next generation. Meanwhile, the answers have a higher chance of survival, which has a higher quality (Talbi, 2009). Based on this, in the proposed solution approach, the local optimal response to the optimal local response with the help of the genetic algorithm.

This research seeks to fill the identified research gap, i.e. not considering the environmental issues arising from queuing in service centers such as delivery and receiving centers, and the transfer of waste and hazardous materials through a muti-objective model of the problem of locating susceptible appropriation Congestion with the M / M / 1 / K queue model with consideration of customer withdrawal prior to arrival, as well as taking into account environmental considerations.

Model Assumptions: The assumptions for the case are as follows:

- Each node can either be the service provider or the customer.
- Each node only allocates to one facility provider (single allocation)
- The triangular inequality is verified in the subject of distances
- Customers' entry follows the Poisson distribution. The service time is exponential, and there is only one service provider in each facility, and the waiting space limit for "k" number of people is defined in the system.
- Providing the service is of "first come-first served" basis, and the queues are considered in the steady-state condition

## 3. Problem Statement

The subject of positioning for fixed crowded facilities is analyzed in the present study. It is assumed in the modeling of the considered subject that each customer makes the required decision before entering the facility, about entering or withdrawal with regards to the present people in the queue. Moreover, due to the importance of environmental problems, in addition to the aims such as minimizing the max. Created queue in the network and withdrawal of the customers, a fitness function is considered in this study that also minimizes the transferring pollution costs between the customers and service providers. According to the descriptions about a case analyzed in this study, a network consisting of "N" nodes is considered. There are "u" distribution or service providing centers for responding to the demands. Each customer can be allocated to a distribution or service providing center. In such conditions, the subject of study is determining the allocation of the service providers in the network to achieve three different aims: (1) since waiting of the customers in the queue is regarded as a critical factor, it can be expected that by minimizing the max. Length of the existing queue in the network, all the service providers face a shorter length of the queue than a max. range; (2) Since the transfer of a matter between a customer and the service provider, or the movement of the customer towards the service provider can have pollutions, such as spreading of carbon monoxide created by the transferring mechanism or emission of hazardous toxic materials during distribution due to the transfer, the second aim is minimizing those types of pollution in the network; (3) Each customer enters with the entry rate "λ" to the service provider center, and when the demand of a customer is sent to that center, there will be the possibility for the service to the customer, id the service provider is free. Otherwise, if the created queue is less than or equal to the permissible range, the customer shall have to be in the queue, and if it is more than the max. Length of the queue, providing the service to the customer is prevented, which leads to adding one person to the number of pf missing customers. Thus, the third aim deals with minimizing the withdrawn customers in the network. Accordingly, the network design should be in such a way to meet the following points: 1) Minimizing the queue length 2) Minimizing the cost of transportation pollution 3) Minimizing the withdrawn customers. In addition to responding to the customers' requirements, designing such a network considers the environmental criteria such as the created carbon dioxide, preventing the emission of toxic materials, and reducing the transportation costs, and it can be originated from the urban, hospital, or industrial waste materials burial areas.

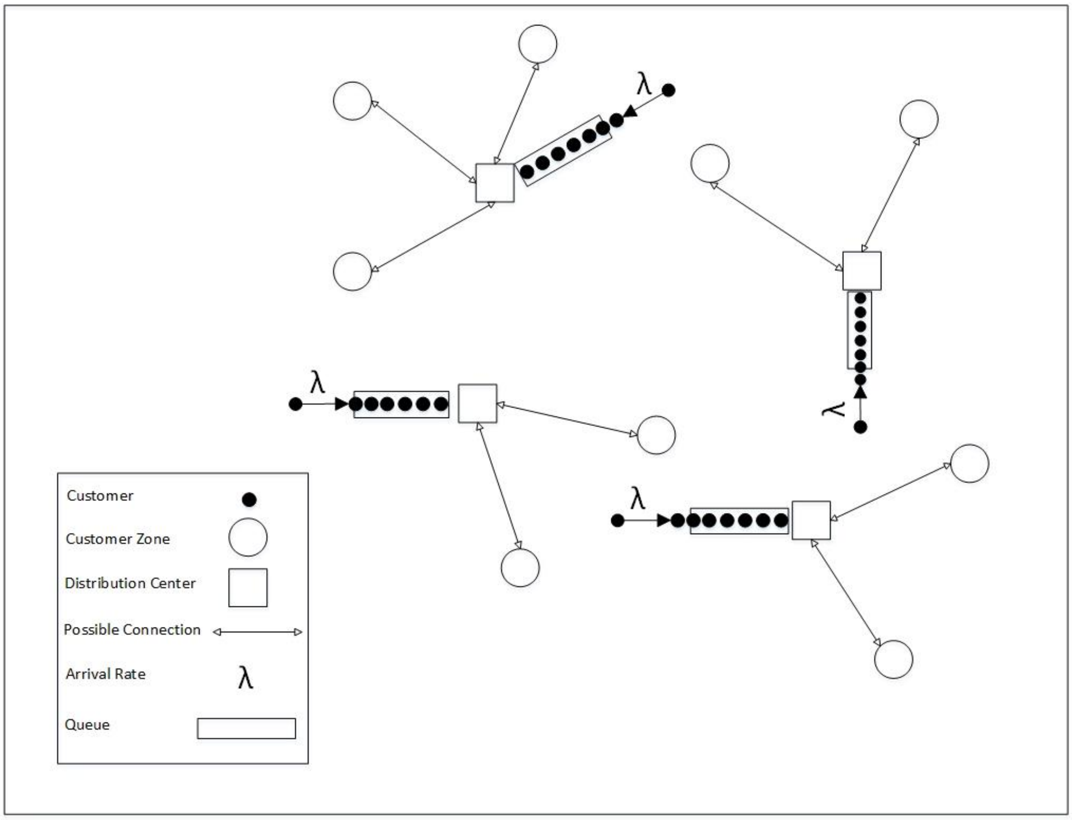



Figure 1 shows a view about the problem

The mathematical model of the problem is as follows:

Set of indices:

$\boldsymbol{i}$: Set of customers ($\boldsymbol{i} \in \{\mathbf{1}, \mathbf{2}, \dots, \boldsymbol{N}\}$: a set of customers).
M: Set of facilities ($m \in \{1,2, \dots, N\}$: a set of facilities).
N: Population of customers in each facility.
Parameters:

$L_q^m$: No. of customers in a queue for the “m” facility (Queuing length)
$L_s^m$: Average no. of the present customers in the “m” facility
$L_b^m$: No. of customers during the service from the “m” facility
$L_q^{max}$: Max. Queue length
$\mu_m$: Service rate of the “m” facility
$\rho_m$: Productivity coefficient of the “m” facility
$K$: Capacity of each facility
$c_{im}$: Cost of the created pollution related to the distance between the customer “i” and the service provider “m”
$t_{im}$: The distance between the customer “i” and the service provider “m” obtained by the points in a “from…..to…..” table in (ORlibrary.com)
Decision variables:

$x_{im}$: If the customer “i” is assigned to facility “m”, it is equal to “1”, and otherwise, it is “0”
$\bar{\bar{\lambda}}_m$: Withdrawn customers from the facility “m”
$\lambda_m$: Set of customers’ rate going to the facility “m”
$\bar{\lambda}_m$: Effective rate of entering to the facility “m”
$\pi_n^m$: Limited probability of the presence of “n” customers in the facility “m”
$u$ : No. of service providers in the system

## 4. Mathematical model

The mathematical model of positioning- multipurpose allocation after considering the environmental concerns is as follows:

$$Min\ f_1 = L_q^{max} \tag{1}$$

$$Min\ f_2 = \sum_i \sum_m c_{im} x_{im} = \sum_i \sum_m \gamma\ t_{im} a^{\beta t_{im}}\ x_{im} \tag{2}$$

$$Min\ f_3 = \sum_{m \in N} \bar{\bar{\lambda}}_m \tag{3}$$

$s.t.$

$$L_q^m \leq L_q^{max} \qquad \forall m \in \{1,2,\dots,N\} \tag{4}$$

$$\lambda_m = \sum_{i}^{N} \lambda_{im} x_{im} \qquad \forall m \in \{1,2,\dots,N\} \tag{5}$$

$$\lambda_m = \bar{\lambda}_m + \bar{\bar{\lambda}}_m \qquad \forall m \in \{1,2,\dots,N\} \tag{06}$$

$$\bar{\lambda}_m = \lambda_m . [1 - \pi_K^m] \qquad \forall m \in \{1,2,\dots,N\} \tag{07}$$

$$\bar{\bar{\lambda}}_m = \lambda_m . \pi_K^m \qquad \forall m \in \{1,2,\dots,N\} \tag{08}$$

$$\sum_{m \epsilon N} x_{im} = 1 \qquad \forall i \in \{1,2,\dots,N\} \tag{09}$$

$$x_{im} \leq x_{mm} \qquad \forall i \in \{1,2,\dots,N\}, m \in \{1,2,\dots,N\} \tag{010}$$

$$\sum_{m \epsilon N} x_{im} = \mathrm{u} \qquad \forall i \in \{1,2,\dots,N\} \tag{11}$$

$$\rho_m = \frac{\lambda_m}{\mu_m} \qquad \forall m \in \{1,2,\dots,N\} \tag{12}$$

$$L_s^m = \sum_{n=0}^{K} n . \pi_K^m \qquad \forall m \in \{1,2,\dots,N\} \tag{13}$$

$$L_b^m = 1 - \pi_0^m \qquad \forall m \in \{1,2,\dots,N\} \tag{14}$$

$$L_q^m = L_s^m - L_b^m \qquad \forall m \in \{1,2,\dots,N\} \tag{15}$$

$$\begin{cases} \pi_0^m = \dfrac{1-\rho_m}{1-\rho_m^{K+1}} & \rho_m \neq 1 \\ \pi_0^m = \dfrac{1}{K+1} & \rho_m = 1 \end{cases} \qquad \forall m \in \{1,2,\dots,N\} \tag{16}$$

$$\begin{cases} \pi_n^m = \frac{(1-\rho_m)\rho_m^n}{1-\rho_m^{K+1}} & \rho_m \neq 1 \\ \pi_n^m = \frac{1}{K+1} & \rho_m = 1 \end{cases} \qquad \forall m \in \{1,2,\dots,N\} \tag{17}$$

$$x_{im} \in \{0,1\} \qquad \forall i \in \{1,2,\dots,N\}, m \in \{1,2,\dots,N\} \tag{18}$$

The fitness function $f_1$ in Equal (1) is considered for minimizing the max. Length of the existing queue in the network. (Bhat, U Narayan 2015)The fitness function $f_2$ in Equal (2) shows the cost of pollution with regards to the distance of the allocated customer “i” to the facility “m” ($t_{im}$), obtained from the position of customer “i” and the facility “m” from a “from…..to…..” table with CAB and AP (ORlibrary.com) data series. The behavior of this function is nonlinear, from the origin to the destination, such that by increasing the distance, the rate of pollution is also increasing, this concept is in the study by (Amin-Naseri et all 2018). In relation to the cost is considered relative to the traffic, such as that$\gamma, a$ and $\beta$ are greater than zero, organizing the behavior of the function for the cost of pollution. The fitness function $f_3$ in Equal (3) is for minimizing the withdrawing customers. Constraint (4) results in max. Length of the queue for the facilities inside the network. Equal (5) determines the total entry rate to each service provider, such that it determines the rates of customer entry to the service provider as the entry rate of the service provider, with regards to allocation of the customers to the service provider. Constraint (6) indicates that the customers’ entry is equal to the sum of the effective rates of ongoing customers and the rate of withdrawn customers. The effective rate of customers’ entrance to each facility is calculated by the aid of equation (7). The rate of outgoing withdrawn customers is calculated by Constraint (8). Constraint (9) guarantees that each customer is allocated to one facility. Constraint (10) guarantees that customer “i” can be allocated to the node “m” when that node is recognized as a facility. Constraint (11) indicates the no. of service providers or facilities in a network. Productivity coefficient is calculated in Constraint (12). Constraint (13) calculates the queue length in each facility together with the people that are being served. Constraint (14) shows the calculation of the no. of customers who are being served. Constraint (15) calculates the queue length in each facility apart from the people who are being served. Constraint (16) calculates the probability in case no one is in the facility. Constraint (17) calculates the probability of the existence of “n” people in each facility, and Constraint (18) shows the range of variables related to the model.

### 4.1. Analysis of the customers’ queue system

#### 4.1.1. M/M/1/K Model

The distance between two successive entries of the customers and the service times in M/M/1/K model are according to exponential distribution (Bhat, U Narayan 2015). Moreover, there is only one service provider in each facility, and the limitation of the waiting space for “K” people is defined in the system. Thus, when the productivity coefficients are equal to$\rho = \frac{\lambda}{\mu}$, the probability functions are defined as follows:

$$\begin{cases} \pi_0 = \dfrac{1-\rho}{1-\rho^{K+1}} & \rho \neq 1 \\ \pi_0 = \dfrac{1}{K+1} & \rho = 1 \end{cases} \tag{19}$$

$$\begin{cases} \pi_n = \dfrac{(1-\rho)\,\rho^n}{1-\rho^{K+1}} & \rho \neq 1 \\ \pi_n = \dfrac{1}{K+1} & \rho = 1 \end{cases} \tag{20}$$

### 4.1.2. Balancing equation in exponential systems: diagram of the proposed model

Is a network is considered with the limitation of “K” people capacity in the system, its thematic diagram for “m” service providers will be as follows:

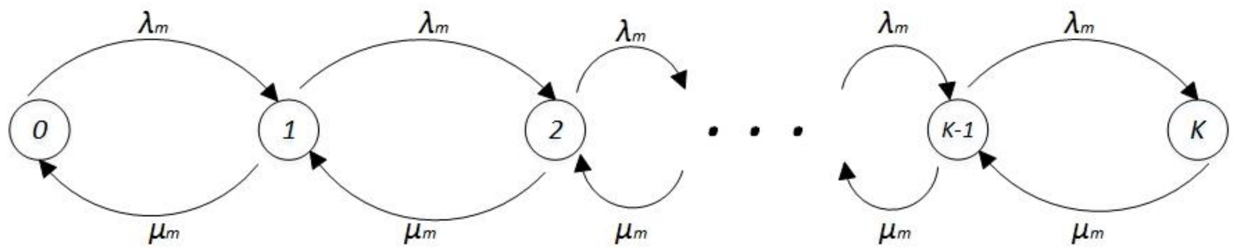



Diagram 1: diagram of the proposed model

### 4.1.3 Normalizing the fitness functions

The “pay-off” table is used for normalizing the fitness functions.

Table 1: “pay-off” table

| | $f_1$ | $f_2$ | $f_3$ |
|---|---|---|---|
| $f_1$ | – | Use optimal solution function 2 ($f_2$) in founction 1 ($f_1$) | Use optimal solution function 3 ($f_3$) in founction 1 ($f_1$) |
| $f_2$ | Use optimal solution function 1 ($f_1$) in founction 2 ($f_2$) | – | Use optimal solution function 3 ($f_3$) in founction 2 ($f_2$) |
| $f_3$ | Use optimal solution function 1 ($f_1$) in founction 3 ($f_3$) | Use optimal solution function 2 ($f_2$) in founction 3 ($f_3$) | – |

Then using the resulted “pay-off” table and relation (21), the normalized values of the fitness functions will be obtained as follows

$$d(x)_n = \frac{f_n^{max} - f(x)_n}{f_n^{max} - f_n^{min}} \tag{21}$$

## 5. Solution approaches

### 5.1. First phase hybrid solution method Based on Genetic Algorithm

Since all the possible solutions for the proposed problem are equal to $\binom{N}{u}u^{N-u}$, the development of the solutions to the question by increasing "N" will be exponential. Thus, the precise solution of the problem is practically impossible for intermediate and large dimensions. Using the genetic algorithm approach in these conditions, the quality answers can be obtained at a logical time. Therefore, we get acquainted in this section with the problem-solving method by the Meta heuristic method of genetic algorithm. The representation of the answers, used operators, and method of generating the primary population is considered at the beginning, and the proposed genetic algorithm is properly described afterward.

#### 5.1.1. Solution representation

All the Meta heuristic algorithms transfer the solution of the problem into an encoded space by the beginning of the solving process, which is called "representation of the answer". In genetics, this structure is referred to as "chromosome". If for instance the number of 10 nodes (N=10) and the number of the distribution center of 3 (u=3) are considered, the structure of the answer for this problem will be in the form of two matrices as in Figure (2).

Facility Matrix

| | | |
|---|---|---|
| 9 | 5 | 1 |

Chromosome

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 5 | 5 | 9 | 5 | 1 | 1 | 9 | 9 | 5 |

Figure (2): Method of representing the answers

#### 5.1.2. Generation of the initial population

For example, for the matrix of service centers, by selecting 3 (no. of distribution centers) random numbers among the numbers 1 to 10 (no. of nodes or customers), the elements of the matrix for the service centers are filled. Regarding the distribution center matrix, the cells become chromosomes, in such a way that the numbers in the matrix of the service centers that are equal to the index of the chromosome matrix are placed in the same chromosome cell, and the other cells that are empty are allocated to the nearest distribution center with respect to their distances. Thus, “N” chromosomes are generated. Figure (3) shows the production method of the primary population.

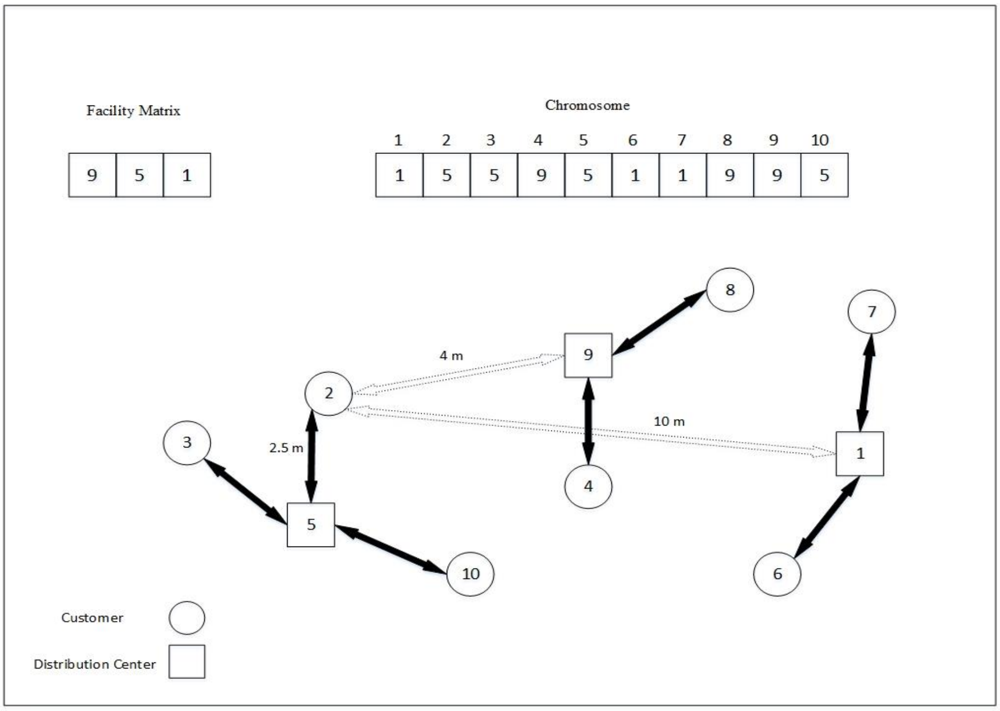



Figure (3): Method of producing the primary population

### 5.1.3. Fitness function

According to relation (21), the normalized fitness of the chromosome is considered in a genetic algorithm, by the time of producing a chromosome. In this regard, the chromosome primarily enters into the fitness function, as a string of numbers. Then, the service points are identified with regards to the placement of the chromosome in the array. To follow, the service providers and the customers allocated to them are entered as the inputs to the first, second, and third fitness functions. The corresponding outputs to each fitness function, including the identified max. Length of the queue in the network ($f_1$), total costs of the network pollutions ($f_2$), and the total withdrawn customers of the network ($f_3$) are entered in relation (21) as the inputs. Finally, the obtained output shows the normalized fitness of the chromosome. This trend continues for other chromosomes with regards to the no. of the considered repetitions for the algorithm, and the better answer is stored each time for the final display.

### 5.1.4. Operators

Two popular operators, namely crossover and mutation operators are used in the genetic algorithm to obtain the new solution (neighboring solution), designed with respect to the chromosome structure. Based on the considered matrix structure for the chromosomes in this regard, the operators are designed as follows:

### 5.1.4.1. Crossover operator

According to the fitness of the chromosomes, m superior ones are selected at each repetition, and $\binom{m}{2}$ children are produced. The method of producing the child is such that a random number between 1 to m (no. of nodes or customers) is selected as the crossover point, and each inherits from its parent relative to the determined crossover point. The way to produce a child is shown in Figure (4).

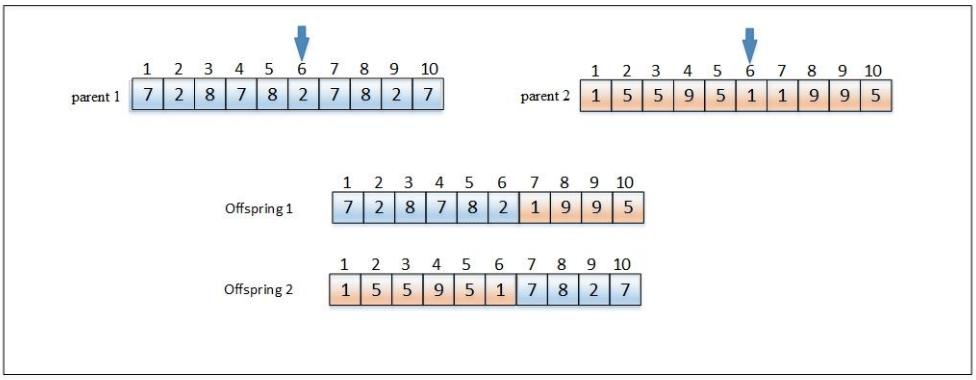



Figure (4): Exhibiting the way to produce a child

### 5.1.4.2. Mutation operator

A chromosome is primarily selected randomly from the previous population to apply this operator, and by considering a probability, one of the following two methods is chosen for the mutation operation.

Method 1: A node is randomly selected among the nodes that are selected as the distribution centers, and interchanged with a node that is allocated as a customer, which is also randomly selected. However, the situation of the points that were connected to the distribution center should be determined in this method. It is dealt with these points in such a way that they are allocated to their nearest distribution center, and finally the produced children are modified with regards to the existing limitations that are determined for the chromosomes. This method is shown in Figure (6).

Method 2: A node is randomly selected among the nodes that are allocated as customers to the distribution centers, and separated from the distribution center to be allocated randomly to the new distribution center. The details of this method are depicted in Figure (7).

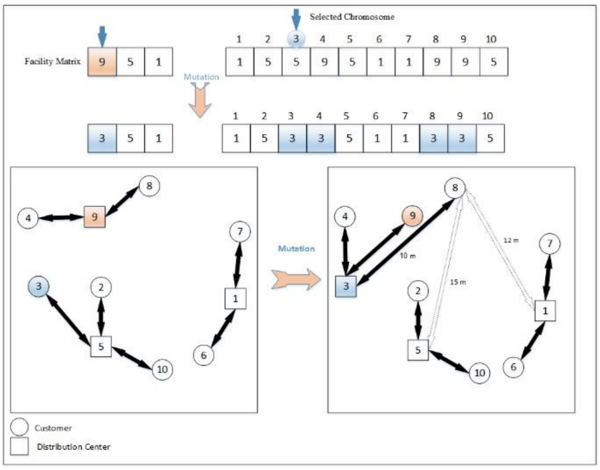



Figure (6): Method of creating the mutation (1st method)

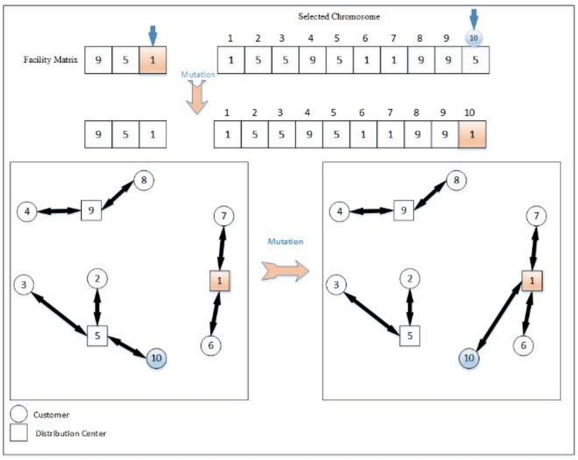



Figure (7): Method of creating the mutation (2nd method)

### 5.1.5. Method of selecting the parent population

For the next stage of the genetic algorithm, 10 superior chromosomes are selected as parents for the next generation, based on the fitness function and the fitness of the chromosomes.

### 5.1.6. Stopping criteria

The last step in the genetic algorithm is analyzing the state of the answer as it was expected by the user. The stopping criteria in this study are defined according to the number of repetitions, and the repetitions are stopped when the algorithm is reaching to that point.

## 5.2. Second phase hybrid solution method Based on Monte Carlo simulation and Genetic Algorithm

The proposed solution is based on Monte Carlo simulation for modeling the discrete probabilistic processes and genetic algorithm for optimization of the problem and it is called GA-MC. Genetic algorithm is discussed in this section first, and the Monte Carlo simulation (referred to GA-MC) will be mentioned next.

### 5.2.1. Genetic algorithm in GA-MC:

The genetic algorithm which has 9 steps is used in GA-MC method.

Step 1 (solution representation): Each Algorithm initially transmits the problem from normal to encoded mode, which is represented by the encoded space representation (Talbi, 2009). In this research, we used a row vector containing zero and one and a number of candidate points (called chromosomes) for positioning servants to show the location of the service providers. If i is the first component of the vector, one indicates that the potential i is chosen as a service provider, and when it is zero, it is expressed that there is no potential source in the service provider.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |

Figure 8: solution display in the coded space

Figure 8 shows the solution in coded space for 10 candidate positions and 3 servants and the positions 3, 5, and 8 are chosen as the servants.

Step 2 (creating the first population): Genetic algorithm is started with an initial population with the number of FP (First Population). In the initial population, for generating each answer, the number of service providers (p) is randomly assigned to component 1. (Figure 9).

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Solution$^{1th}$ | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| Solution$^{2th}$ | 0 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 | 0 |
| ⋮ | | | | | | | | | | |
| Solution$^{FPth}$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |

Figure 9: First population

Step 3 (Determine population fitness): in this step, each of the answers in the population is used in the Monte Carlo simulation based on equations (1), (2) and (3) to evaluate the suitability of that population.

Step 4 Selecting the Elite Generation: At this point in the Genetic Algorithm, the best answer is selected in terms of the fitness function (E).

Step 5 (crossover and child production): in this step, each pair from the best E solutions (chromosomes) is merged by single point crossover operator. For this purpose, the operator of a Single point crossover is used. Under this operator, the parent chromosomes are broken down into two pieces (from a chosen point by chance) and the right part of the parent chromosome and left part of the parent chromosome are gathered to create an offspring chromosome (Figure 10).

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Parent$^{1th}$ | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 | 0 | 0 |
| Parent$^{2th}$ | 0 | 1 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| Offspring | 0 | 0 | 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |

Figure 10: Combining two parent responses under the operation of a Single point crossover

In order for the offspring chromosome to be acceptable, the number 1 should not be greater than the desired number (P). The offspring chromosome will be removed if it has more than (P) components of number 1 and the single point crossover will be repeated for another chosen point by chance. There will be $\binom{E}{2}$ new offspring in this step.

Step 7 (Determine suitability of offspring): in this step, every offspring is used in the Monte Carlo simulation based on equations (1), (2) and (3) to evaluate the suitability of them.

Step 8 (formation of the new generation): By gathering the offspring and the elite generation, a new generation is formed.

Step 9: Checking the Stop Condition: At this stage, it is examined whether the genetic algorithm is repeated in MAX order. If the answer is yes, the algorithm stops and gives the best answer to the elite genre, otherwise it goes to step 4.

### 5.2.2. Monte Carlo simulation in GA-MC:

As mentioned in the previous section, the evaluation of the solutions of the genetic algorithm is done by the Monte Carlo simulation proposed by GA-MC. The steps of the Monte Carlo simulation are as follows:

Step 0 – take $\forall i = 1,2, \dots, p, q_i \leftarrow 0$ and $f_3 \leftarrow 0$

Step 1 – do steps 2 to 10 for IT times

Step 2 – while $t < t_{max}$ do steps 3 to 9

Step 3 – create a customer by chance based on an exponential distribution with rate λ

Step 4 – create a random position for the customer based on two variables -Gaussian distribution with means of $[\mu_1, \mu_2]$ and standard deviations of$[\sigma_1, \sigma_2]$.

Step 5 – Assign the client to the nearest servant (suppose i) according to Euclidean norm.

Step 6 - If the service provider i is free to put it in a busy state otherwise, go to step 7

Step 7 - If the length of the queuing service i ($q_i$) is less than the queuing capacity ($Cap_i$) ($q_i < Cap_i$), add one to the queue length ($q_i \leftarrow q_i + 1$) and go to step 9.

Step 8- If the length of the queue of the serving i ($q_i$) is greater than or equal to the queuing capacity ($Cap_i$) ($q_i \geq Cap_i$), add one to the deniers, ie ($f_3 \leftarrow f_3 + 1$) and go to step 9.

Step 9 - Specify the waiting time and queue length for the ith servant.

Step 10- Calculate the values of $f_{1k}$ and $f_{2k}$ in repeat Kth.

Step 11- Calculate the algorithm's repeat order according to equations (1), (2) and (3)$f_1$, $f_2$, and $f_3$ for the ITth iteration of the algorithm.

## 6. NUMERICAL RESULTS

### 6.1. First phase Numerical results

Since solving the proposed model of the positioning-allocation in the queuing system by considering the environmental aspects in intermediate and large dimensions is among the complicated problems, and the precise solutions are impractical due to the limitations such as long-time solutions, the Meta heuristic method of genetic algorithm is used to solve this problem in intermediate and large dimensions. It is to note that the genetic algorithm is encoded in MATLAB R2016a software to solve the proposed model, and all the calculations are taken to a CPU G3250 3.20 GHz / RAM 4GB system.

Figure (11) illustrates the trend belonging to each execution before the 500$^{th}$ repetition, and leading of the trends to value shows that the graph has an acceptable convergence. In other words, the algorithm acts robustly.

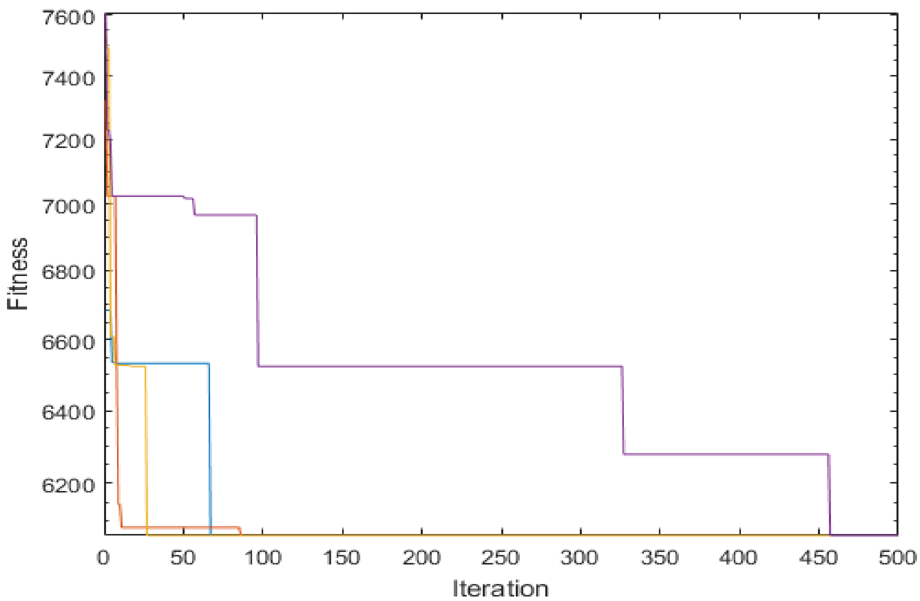



Figure 11: convergence figure

#### 6.1.1. Results of the solution in medium and large scale

In Table (2), the column (N) indicates the number of nodes, column (u) shows the number of distributors or service providers, column (Best) shows the best chromosome, column ($f_1$) indicates the values of the first fitness function (minimizing the max. length of the queue in the network), column ($f_2$) depicts the values of the 2$^{nd}$ fitness function (min. cost of pollution in the network), column ($f_3$) indicates the values of the 3$^{rd}$ fitness function (min. lost demand in the network), column ($d_1(x)$) shows the normalized values of the 1$^{st}$ fitness function, while columns ($d_2(x)$) and ($d_3(x)$) show the normalized values of the 2$^{nd}$ and 3$^{rd}$ fitness functions, respectively, and column (D) shows the normalized values of the fitness function resulted from relation (21). Column (CPU Time) indicates the time of solving the examples in two methods. Finally, the difference between the obtained answers by the precise solution and Meta heuristic methods are brought in column (Gap rate). According to the difference, the obtained answers between the precise solution and Meta heuristic methods in the problem with small dimensions are less than 1%. Thus, it can be said that the obtained answers from the proposed genetic algorithm for the problems in intermediate and large dimensions are considered to be with good quality. In Table (2), the problem is solved for 20 and 25 nodes and by using CAB data series, and in Table (3), it is solved for 20 and 40 nodes and by using AP data series with the genetic Meta heuristic algorithm.

Table 2: Numerical results of the first phase of the solution

| $N$ | $u$ | Methods | $Best$ | $f_1$ | $f_2$ | $f_3$ | $d_1(x)$ | $d_2(x)$ | $d_3(x)$ | $D$ | $CPU\ Time$ | Gap %=$(\frac{ES-Meta}{Meta})\times100$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 8 | 2 | Exhaustive Search | [5.6.6.5.5.6.5.5] | 6.8043 | 0.76668 | 97 | 0.99926 | 0.65342 | 1 | 0.91831 | 2 | 0 |
| | | Genetic Algorithm | [5.6.6.5.5.6.5.5] | 6.8043 | 0.76668 | 97 | 0.99926 | 0.65342 | 1 | 0.91831 | 0.637 | |
| | 3 | Exhaustive Search | [5.6.6.5.5.6.7.7] | 6.7222 | 0.44422 | 89 | 0.66341 | 0.83786 | 0.74998 | 0.86928 | 6 | 0 |
| | | Genetic Algorithm | [5.6.6.5.5.6.7.7] | 6.7222 | 0.44422 | 89 | 0.66341 | 0.83786 | 0.74998 | 0.86928 | 0.588 | |
| | 4 | Exhaustive Search | [5.6.3.6.5.6.7.7] | 6.6429 | 0.3396 | 80 | 0.81238 | 0.83534 | 0.99761 | 0.93472 | 5 | 0 |
| | | Genetic Algorithm | [5.6.3.6.5.6.7.7] | 6.6429 | 0.3396 | 80 | 0.81238 | 0.83534 | 0.99761 | 0.93472 | 0.558 | |
| 9 | 2 | Exhaustive Search | [5.6.6.6.5.6.5.5.6] | 7.8305 | 0.75563 | 110 | 0.95255 | 0.69957 | 1 | 0.92427 | 1 | 0 |
| | | Genetic Algorithm | [5.6.6.6.5.6.5.5.6] | 7.8305 | 0.75563 | 110 | 0.95255 | 0.69957 | 1 | 0.92427 | 0.822 | |
| | 3 | Exhaustive Search | [5.6.6.5.5.6.7.7.6] | 7.7429 | 0.61158 | 102 | 0.86108 | 0.71594 | 0.75 | 0.87597 | 45 | 0 |
| | | Genetic Algorithm | [5.6.6.5.5.6.7.7.6] | 7.7429 | 0.61158 | 102 | 0.86108 | 0.71594 | 0.75 | 0.87597 | 0.8 | |
| | 4 | Exhaustive Search | [5.3.3.6.5.6.7.7.6] | 7.6539 | 0.36911 | 93 | 0.82822 | 0.86948 | 0.85745 | 0.92375 | 187 | 0 |
| | | Genetic Algorithm | [5.3.3.6.5.6.7.7.6] | 7.6539 | 0.36911 | 93 | 0.82822 | 0.86948 | 0.85745 | 0.92375 | 1 | |

Table 2: Numerical results of the first phase of the solution (continued)

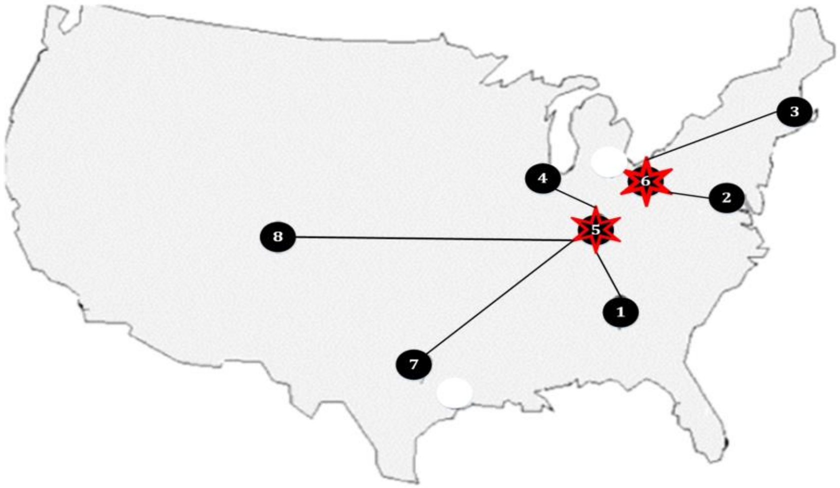
| $N$ | $u$ | Methods | $Best$ | $f_1$ | $f_2$ | $f_3$ | $d_1(x)$ | $d_2(x)$ | $d_3(x)$ | $D$ | $CPU\ Time$ | Gap % |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 2 | Exhaustive Search | [10.6.6.6.6.6.10.10.6.10] | 8.8413 | 0.85058 | 124 | 0.94881 | 0.75128 | 1 | 0.93699 | 26 | 0 |
| | | Genetic Algorithm | [10.6.6.6.6.6.10.10.6.10] | 8.8413 | 0.85058 | 124 | 0.94881 | 0.75128 | 1 | 0.93699 | 3 | |
| | 3 | Exhaustive Search | [5.5.6.6.5.6.10.10.10.6.10] | 8.7692 | 0.59287 | 115 | 0.75961 | 0.62518 | 1 | 0.87355 | 875 | 0 |
| | | Genetic Algorithm | [5.5.6.6.5.6.10.10.10.6.10] | 8.7692 | 0.59287 | 115 | 0.75961 | 0.62518 | 1 | 0.87355 | 5 | |
| | 4 | Exhaustive Search | [5 3 3 6 5 6 10 10 6 10] | 8.7059 | 0.46126 | 106 | 0.7247 | 0.77016 | 0.99999 | 0.90436 | 9414 | 0.4246 |
| | | Genetic Algorithm | [5 3 3 6 5 6 10 10 6 10] | 8.7188 | 0.43959 | 106 | 0.6548 | 0.81388 | 0.99945 | 0.90052 | 3 | |
| 11 | 2 | Exhaustive Search | [10 6 6 6 6 6 10 10 6 10 10] | 9.8592 | 0.84941 | 134 | 0.77867 | 0.74628 | 1 | 0.90841 | 199 | 0 |
| | | Genetic Algorithm | [10 6 6 6 6 6 10 10 6 10 10] | 9.8592 | 0.84941 | 134 | 0.77867 | 0.74628 | 1 | 0.90841 | 5 | |
| | 3 | Exhaustive Search | [5 6 6 5 5 6 10 10 6 10 5] | 9.7959 | 0.67037 | 125 | 0.77679 | 0.80502 | 1 | 0.92195 | 15148 | 0.5608 |
| | | Genetic Algorithm | [5 6 6 5 5 6 10 10 6 10 5] | 9.7727 | 0.73526 | 125 | 0.97053 | 0.66234 | 1 | 0.91678 | 7 | |
| 12 | 2 | Exhaustive Search | [10 6 6 6 6 6 10 10 6 10 10 10] | 10.8734 | 1.1936 | 150 | 0.70829 | 0.74485 | 1 | 0.89525 | 5376 | 0 |
| | | Genetic Algorithm | [10 6 6 6 6 6 10 10 6 10 10 10] | 10.8734 | 1.1936 | 150 | 0.70829 | 0.74485 | 1 | 0.89525 | 5 | |

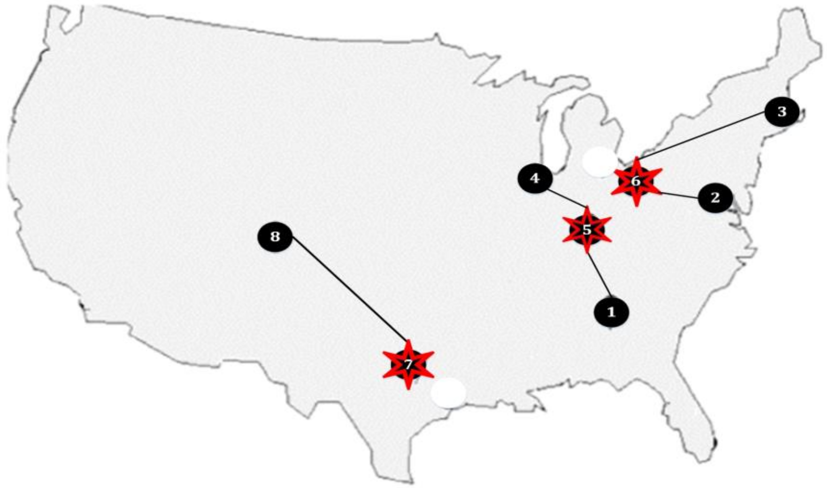
Table 2: Numerical results of the first phase of the solution (continued)

| GA CAB Data | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $u$ | $Best$ | $f_1$ | $f_2$ | $f_3$ | $d_1(x)$ | $d_2(x)$ | $d_3(x)$ | $D$ | $CPU\ Time$ |
| 20 | 2 | [10,6,6,6,6,6,10,10,6,10,6 10,10,10, 6,10,6,6,10,6] | 18.9254 | 2.3238 | 267 | 0.99152 | 0.84972 | 1 | 0.96672 | 4.127 |
| | 3 | [5 6 6 5 5 6 10 10 6 10 5 10 5 5 5 10 6 6 10 6] | 18.8936 | 1.9646 | 258 | 0.93404 | 0.55256 | 1 | 0.87908 | 4.865 |
| | 4 | [5 5 6 5 5 6 10 12 6 10 5 12 10 10 5 10 6 6 12 6] | 18.8701 | 1.4724 | 249 | 0.79667 | 0.71898 | 1 | 0.90476 | 6.318 |
| $25$ | $2$ | [6 6 6 6 6 6 10 10 6 10 10 10 10 10 6 10 6 6 10 6 10 10 6 10 6] | 23.9444 | 3.9709 | 353 | 0.97884 | 0.68035 | 1 | 0.9229 | 4.989 |
| | 3 | [25 25 25 5 5 5 5 12 5 12 5 12 5 25 5 5 25 25 12 25 5 12 12 25 25] | 23.9286 | 2.5844 | 345 | 0.71978 | 0.72436 | 0.8 | 0.86305 | 5.549 |
| | 4 | [6 25 25 6 6 6 10 12 6 10 10 12 10 25 25 10 25 25 12 6 6 12 12 10 25] | 23.8913 | 2.0567 | 334 | 1 | 0.83076 | 1 | 0.9636 | 6.975 |

Table 2: Numerical results of the first phase of the solution (continued)

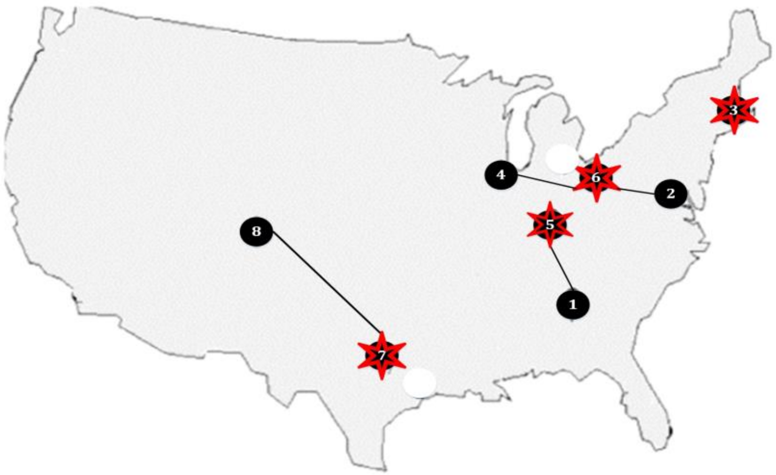
| GA AP Data | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $N$ | $u$ | $Best$ | $f_1$ | $f_2$ | $f_3$ | $d_1(x)$ | $d_2(x)$ | $d_3(x)$ | $D$ | $CPU\ Time$ |
| 25 | 2 | [6 6 6 25 25 6 6 6 25 25 25 25 25 6 6 6 6 25 25 25 6 6 25 25 25] | 23.9438 | 90.3952 | 353 | 1 | 0.99677 | 1 | 0.99935 | 5.951 |
| | 3 | [6 6 6 10 10 6 6 10 10 10 6 6 10 10 10 6 25 25 25 10 25 25 25 25 25] | 23.916 | 1.4346 | 343 | 0.95531 | 0.88265 | 1 | 0.96868 | 5.795 |
| | 4 | [6 6 6 25 10 6 6 10 10 10 6 6 25 10 10 22 22 22 25 10 22 22 25 25 25] | 23.8889 | 0.5933 | 335 | 0.98765 | 0.87199 | 0.91667 | 0.95857 | 6.999 |
| 40 | 2 | [6 25 6 6 6 6 6 6 25 25 25 6 6 6 6 6 25 25 25 25 6 6 6 6 25 25 25 25 25 25 6 6 25 25 25 25 25 25 25 6] | 38.9644 | 0.31375 | 562 | 0.97707 | 0.90439 | 1 | 0.9767 | 7.153 |
| | 3 | [10 10 10 6 6 6 6 6 10 10 10 10 6 6 6 6 25 10 10 10 10 6 6 6 25 25 25 25 25 25 10 6 10 25 25 25 25 25 25 25] | 38.9465 | 0.18286 | 552 | 1 | 0.88925 | 1 | 0.9768 | 8.051 |
| | 4 | [5 12 5 5 5 5 5 5 25 12 12 12 12 12 12 12 25 25 25 12 38 12 38 12 25 25 25 38 38 38 38 38 25 25 25 38 38 38 38 38] | 38.9427 | 0.11335 | 545 | 1 | 0.8173 | 0.71429 | 0.91318 | 8.764 |

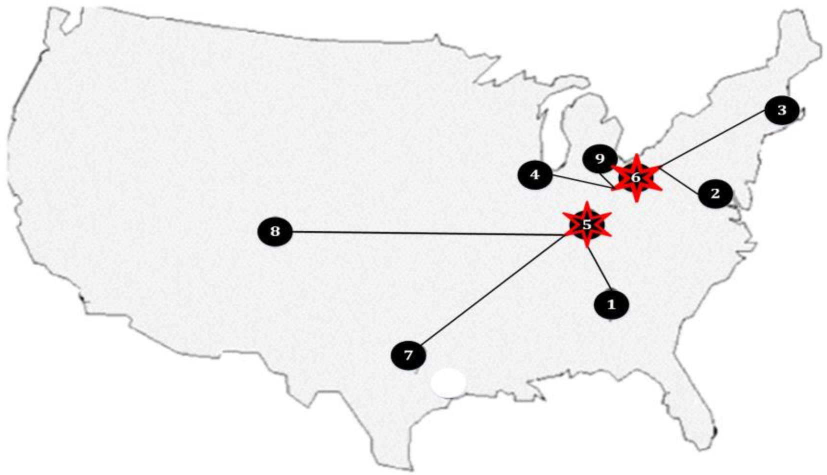



(a): node=8 DC=2

(b): node=8 DC=3

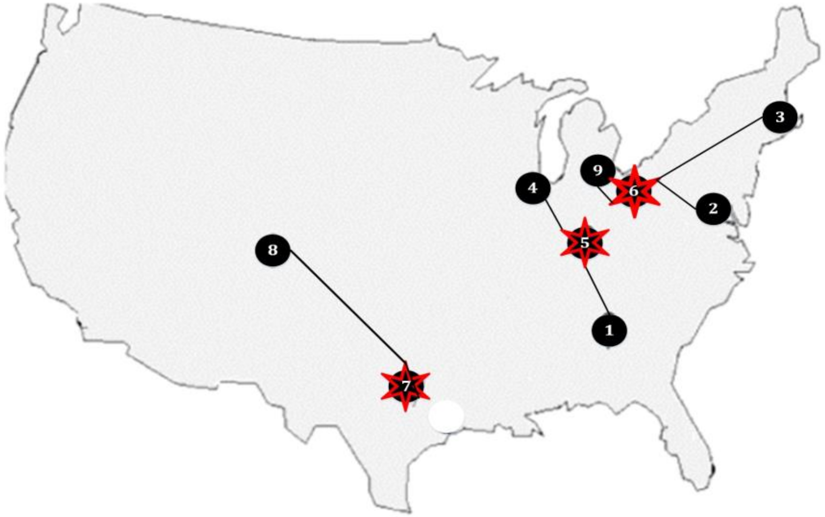


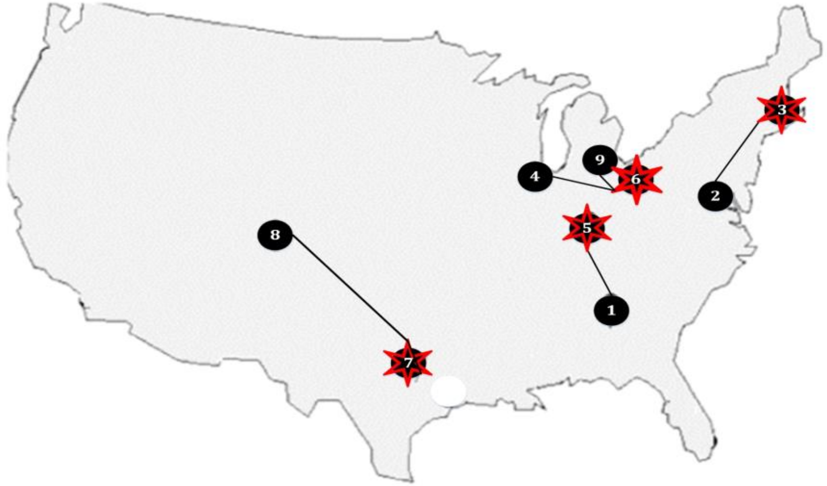

(c): node=8 DC=4

(d): node=9 DC=2

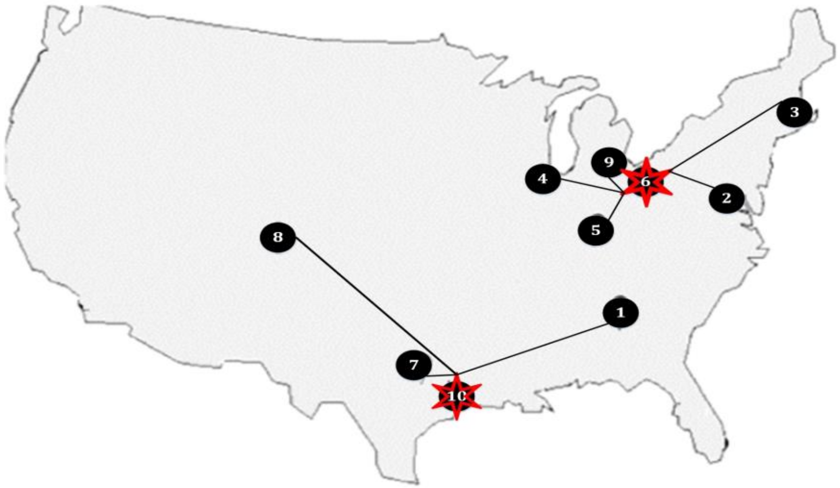
Figure 12: The location structure of the allocation of service centers in the first phase of the solution



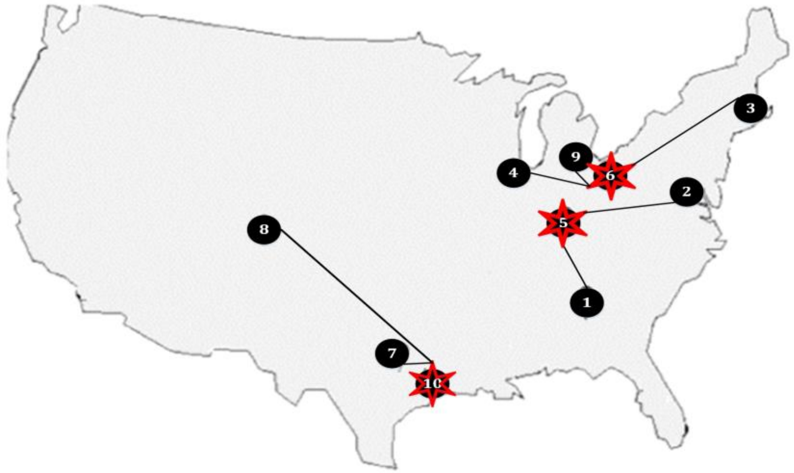

(e): node=9 DC=3




(f): node=9 DC=4

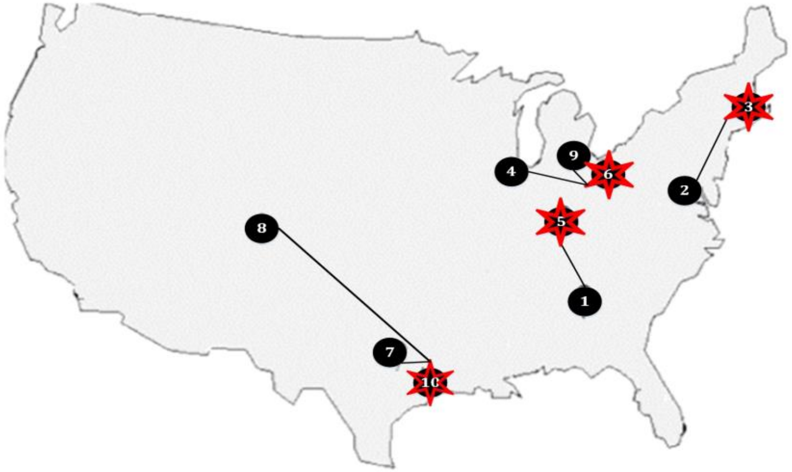



(g): node=10 DC=**2**

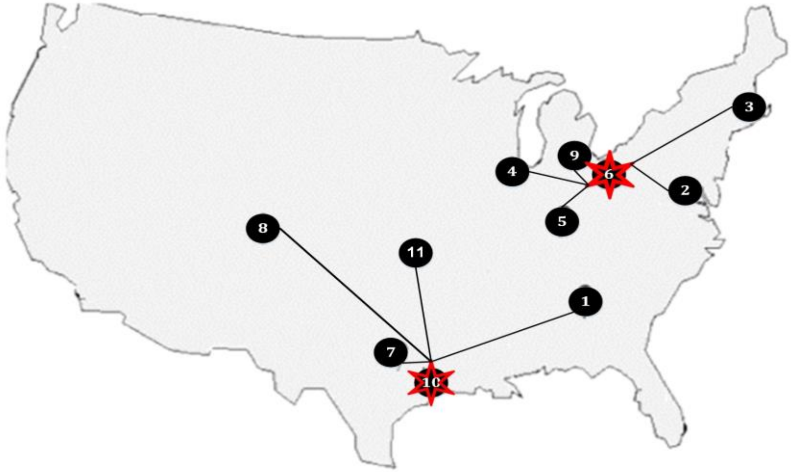



(h): : node=10 DC=3

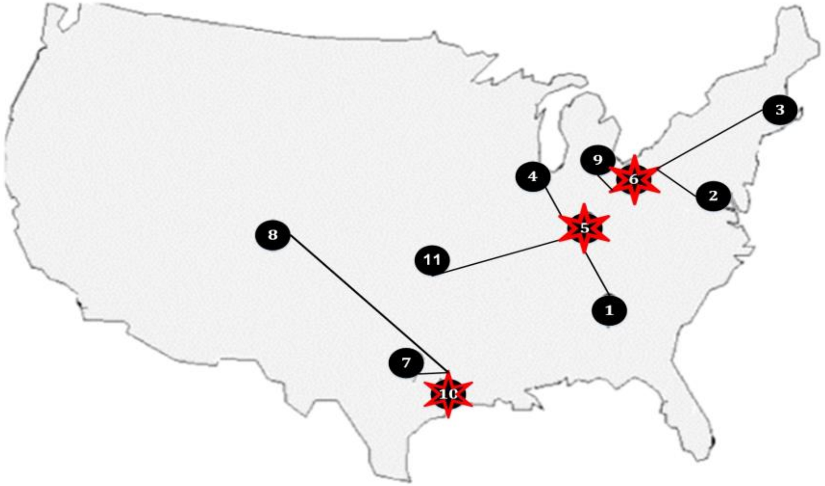
Figure 12: Location structure of the allocation of service centers in the first phase of the solution (continued)



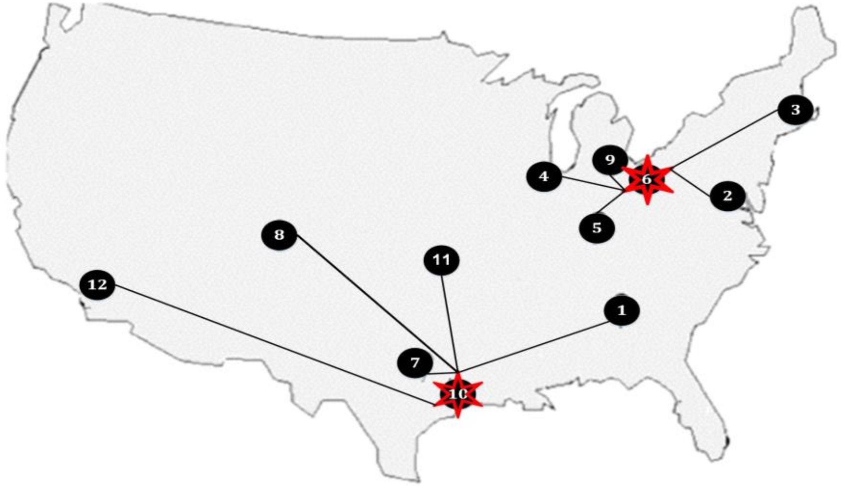

(i): node=10 DC=4




(j): node=11 DC=2




(k): node=11 DC=3




(l): node=12 DC=2

Figure 12: Location structure of the allocation of service centers in the first phase of the solution (continued)

In Tables 2, the column "Best chromosome optimized answer" is shown and the numbers in it indicate which points are selected as the service providers, and their allocations are also specified with regards to their positions in the chromosome, such that for example if the number "5" is the first number of the chromosome string, it shows that the point "5" is selected as the service provider and customer "1" is allocated to it. Therefore, the problem is solved in the first phase from the viewpoint of the allocation position. Used data is from CAB data series. In figure 12, the following illustrations show the places that are selected as the service providers and their states of allocation are specified.

### 6.2. Second phase numerical solution

In this phase, the hypotheses of the numerical example including the statistical distribution of the position of the appearance of customers, classification of the positions based on the kind of servants and the speed of services in different positions are explained first. Next, the numerical results in three different scenarios are shown and sensitivity analyzed.

To define the probable position of the servants, the x-axis contains 100 points and the y-axis consists of 100 points with 10 intervals, as shown in Figure 13.

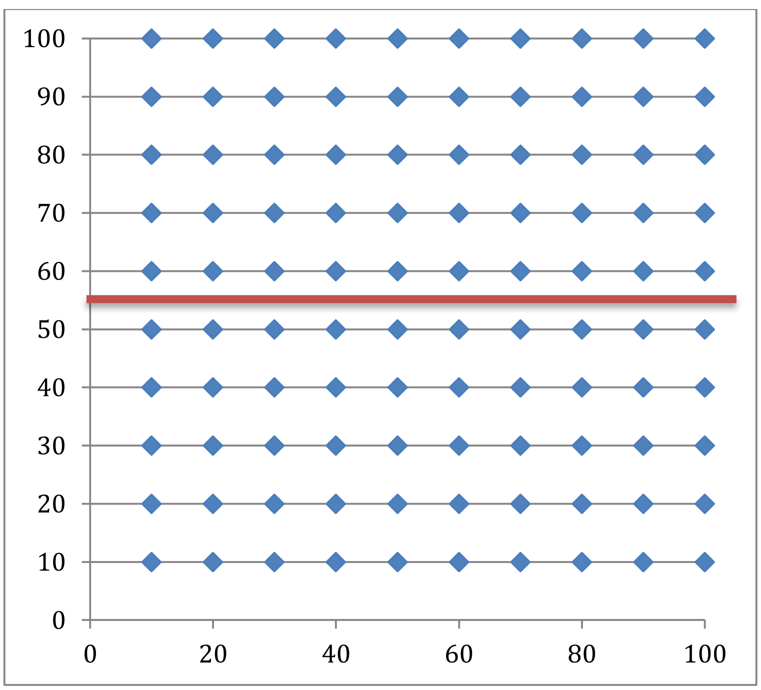



Figure 13: Potential Location of Service Providers

As shown in Figure 13, the service area is divided into two northern regions (above the red line) and the southern (bottom of the red line). Serving speed and maximum queue length are considered for these two different regions. This is true in the real world due to financial and geographical constraints. Customers' entry rates are defined as exponential distribution with averages equal to and equal to $\lambda = 1$. It should be noted that the uncertainties of the problem include the appearance of customers, the arrival rate of customers, and the servicing standards of service providers, which are considered in three scenarios in the problem as follows.

#### 6.2.1. First scenario

The first scenario assumes that the arrival place of customers is randomized with a normal distribution of two variables with $\mu = [50, 50]$ and $\sigma = [250,250]$, as shown in Figure 14.

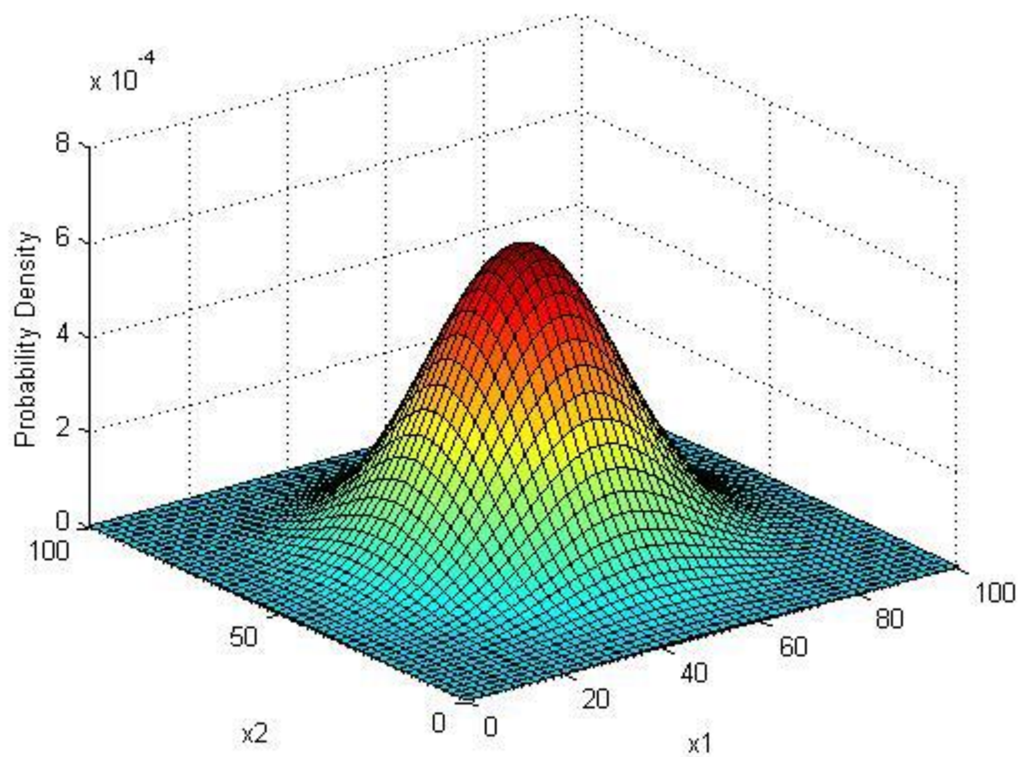



Figure 14: How to enter customers with a normal binary variance with $\mu = [50, 50]$ and $\sigma = [250,250]$

Table 3 shows the results of the first scenario. The first column is equivalent to the number of service providers defined as $n \in \{2, 3, 4, \text{ and } 5\}$. The second and third columns represent, respectively, the average customer service time in the potential areas for the southern ($\mu\_1$) and northern ($\mu\_2$) service areas. It should be noted that the service time has an exponential distribution with mean $\mu$. Also, the fourth and fifth pillars of limited capacity are defined for service providers located in the southern regions (Cap1) and North (Cap2) area of customer service. The value of the objective function is aggregated and the first to third fitness are shown in columns 6 to 9, respectively, and the last column represents the time of the proposed simulation algorithm. The customer entry rate is defined as an exponential distribution with mean λ = 1.

Table 3: Results of the first scenario

| $n$ | $\mu_1$ | $\mu_2$ | $Cap_1$ | $Cap_2$ | $f_1$ | $f_2$ | $f_3$ | $CPU$ |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 4 | 8 | 2 | 28.24697 | 1.624311 | 447 | 70.031 |
| | 2 | 4 | 2 | 8 | 20.77918 | 1.106885 | 228 | 70.129 |
| | 4 | 2 | 8 | 2 | 27.05659 | 1.254386 | 162 | 70.123 |
| | 4 | 2 | 2 | 8 | 31.96758 | 1.65596 | 419 | 71.936 |
| 3 | 2 | 4 | 8 | 2 | 27.51101 | 1.72596 | 180 | 72.067 |
| | 2 | 4 | 2 | 8 | 25.59412 | 1.031769 | 138 | 72.266 |
| | 4 | 2 | 8 | 2 | 29.98182 | 0.940874 | 104 | 73.447 |
| | 4 | 2 | 2 | 8 | 30.90423 | 1.666109 | 203 | 73.554 |
| 4 | 2 | 4 | 8 | 2 | 36.5672 | 1.814772 | 104 | 74.111 |
| | 2 | 4 | 2 | 8 | 23.67559 | 0.977305 | 121 | 74.136 |
| | 4 | 2 | 8 | 2 | 31.39547 | 0.924209 | 115 | 74.169 |
| | 4 | 2 | 2 | 8 | 37.35364 | 1.536349 | 129 | 74.273 |
| 5 | 2 | 4 | 8 | 2 | 35.56771 | 1.6914 | 98 | 75.320 |
| | 2 | 4 | 2 | 8 | 28.43265 | 0.797853 | 69 | 75.346 |
| | 4 | 2 | 8 | 2 | 25.18065 | 1.129889 | 74 | 75.231 |
| | 4 | 2 | 2 | 8 | 32.66818 | 1.471411 | 79 | 75.642 |

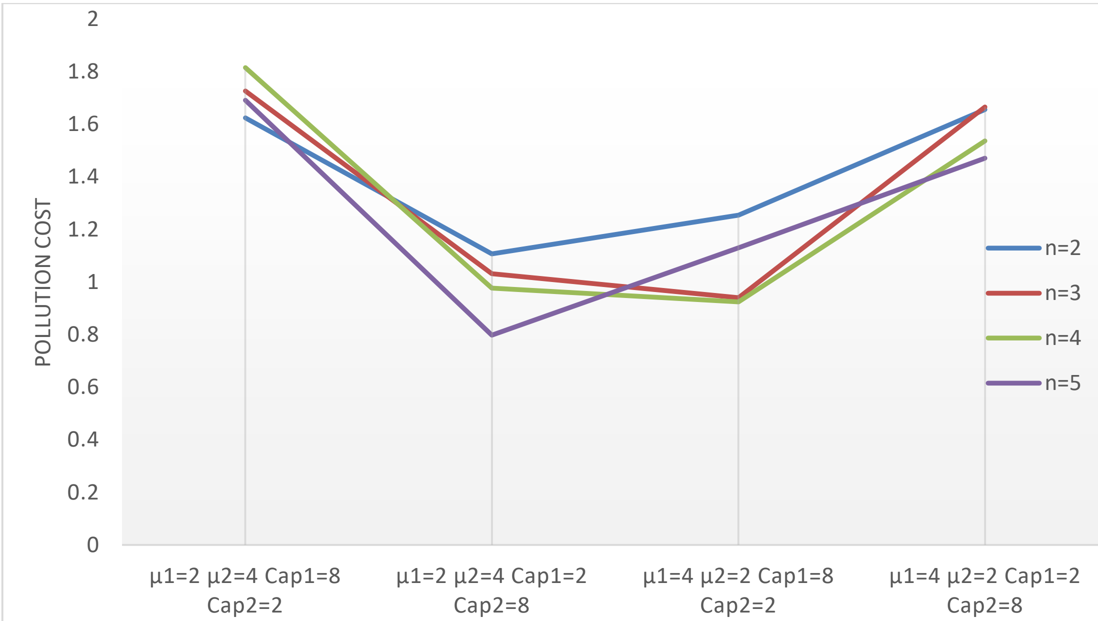



Figure 15: The behavior of the pollution cost function versus changes in the number of service providers, service speed and capacity (first scenario)

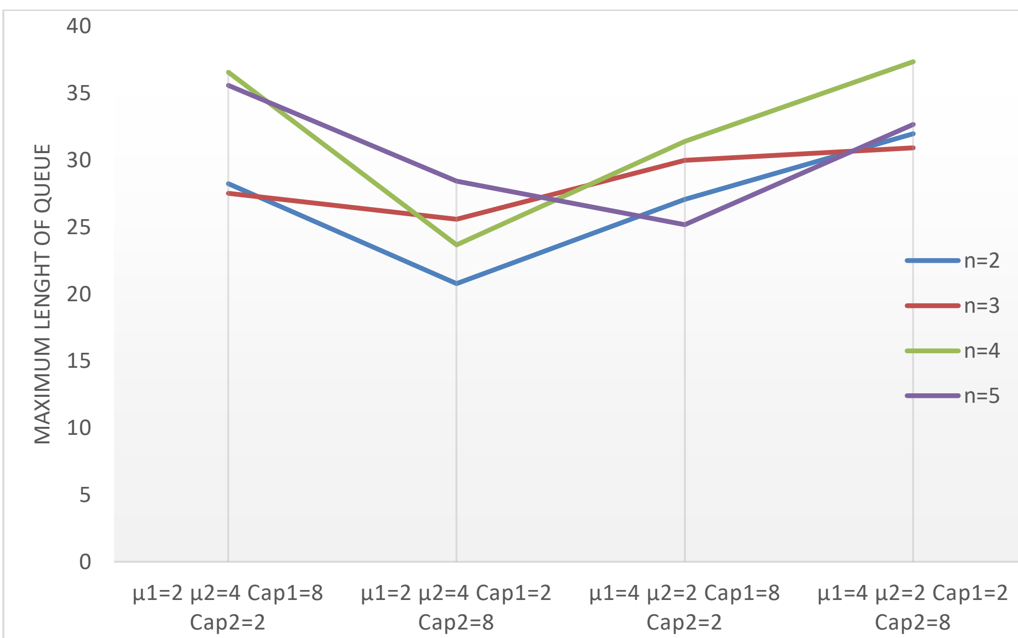



Figure 16: Maximum function of the customer's queue length in relation to changes in the number of employees, service speed and capacity (first scenario)

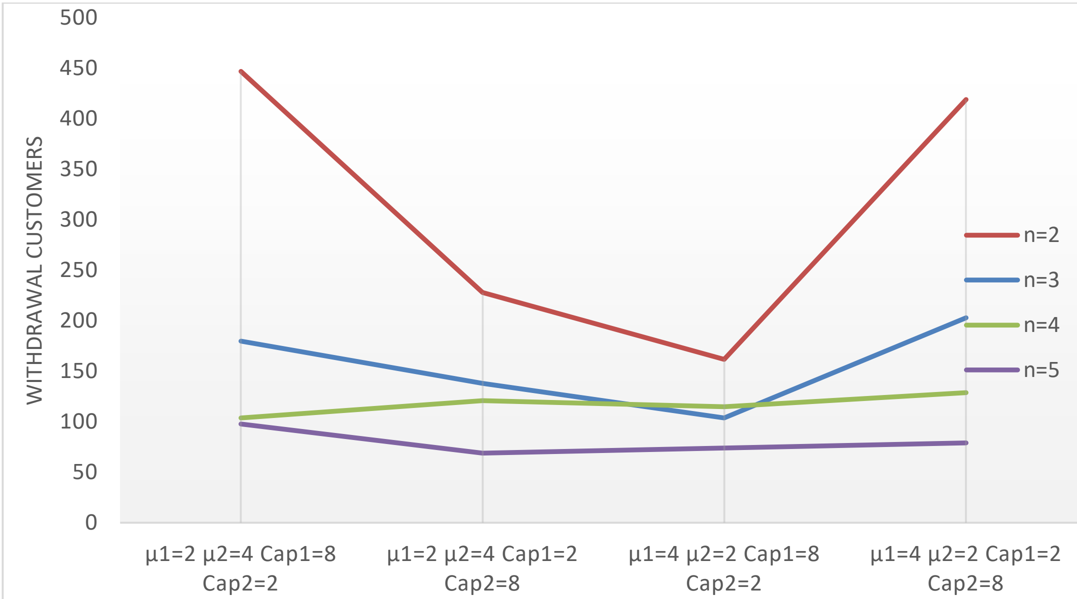



Figure 17: Number of customers' withdrawal behavior versus changes in the number of service providers, service speed and capacity (first scenario)

The results of Table 3 show:

- When a higher queuing capacity (Cap = 8) is assigned to a high service user (μ = 4), the mean length of the queue decreases, so the cost of pollution for the entire system decreases. Because the service provider is slowly expecting a higher queue. This is true of the servitor in the northern or southern region. Figure 15 illustrates the conditions described.
- When the service is slowed down (μ = 4), the queue capacity is assigned (Cap = 2) the queue length increases. Figure 16 shows this.
- With the increase in the number of service providers, the number of customers dropping in each of the four cases is reduced. Also, when the service provider is high (μ = 4), the queue is assigned a higher queue (Cap = 8), the number of customers for the denial of service for the entire system decreases. This is shown in Figure 17.

### 6.2.2 Second scenario

As a second scenario, it is assumed that the customer entry point follows a randomized, normal distribution of two variables with μ = [50, 25] and σ = [250,250] as follows. Other assumptions are in accordance with the first scenario.

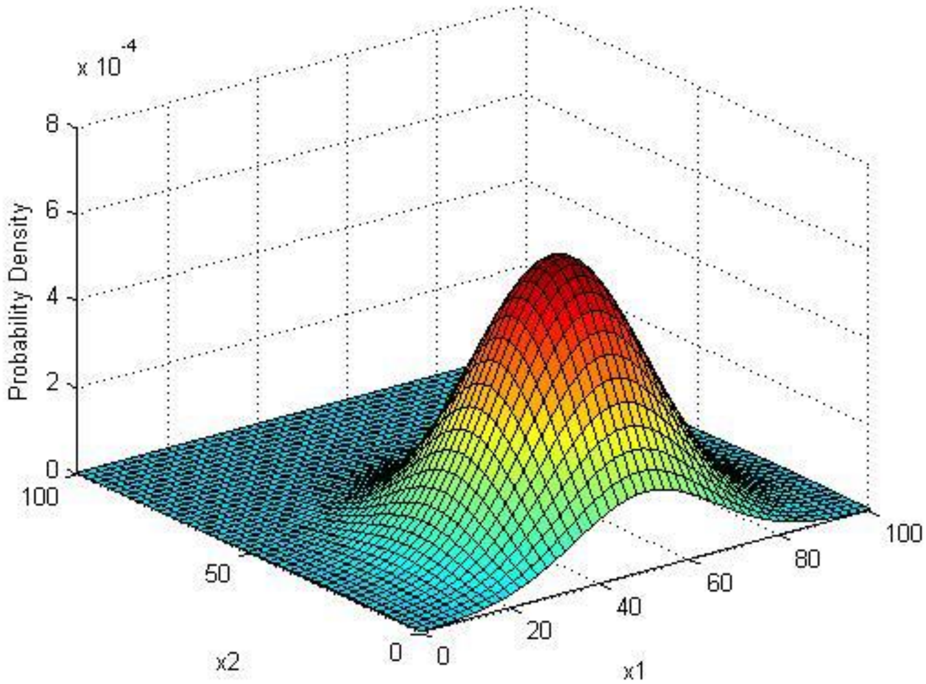



Figure 18: How to enter customers with a normal two-variable distribution with μ = [50, 25]
[250,250] = and σ

Table 4: Results of the second scenario

| $n$ | $\mu_1$ | $\mu_2$ | $Cap_1$ | $Cap_2$ | $f_1$ | $f_2$ | $f_3$ | $CPU$ |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 4 | 8 | 2 | 26.05224 | 1.516338 | 595 | 69.663 |
| | 2 | 4 | 2 | 8 | 23.34028 | 1.161446 | 196 | 70.589 |
| | 4 | 2 | 8 | 2 | 32.02575 | 1.150391 | 265 | 70.681 |
| | 4 | 2 | 2 | 8 | 27.16131 | 1.589485 | 466 | 70.256 |
| 3 | 2 | 4 | 8 | 2 | 41.46523 | 1.447716 | 280 | 72.189 |
| | 2 | 4 | 2 | 8 | 23.09957 | 0.980854 | 91 | 72.413 |
| | 4 | 2 | 8 | 2 | 40.43771 | 1.163974 | 172 | 72.506 |
| | 4 | 2 | 2 | 8 | 41.97995 | 1.828884 | 180 | 72.932 |
| 4 | 2 | 4 | 8 | 2 | 37.11908 | 1.554243 | 27 | 74.811 |
| | 2 | 4 | 2 | 8 | 24.10316 | 0.801381 | 45 | 74.052 |
| | 4 | 2 | 8 | 2 | 39.10371 | 0.956715 | 166 | 74.755 |
| | 4 | 2 | 2 | 8 | 34.70884 | 1.538009 | 215 | 74.893 |
| 5 | 2 | 4 | 8 | 2 | 32.10045 | 1.240592 | 20 | 74.820 |
| | 2 | 4 | 2 | 8 | 23.69132 | 0.677974 | 46 | 75.330 |
| | 4 | 2 | 8 | 2 | 41.35926 | 0.87386 | 87 | 75.174 |
| | 4 | 2 | 2 | 8 | 39.54114 | 1.50116 | 167 | 74.655 |

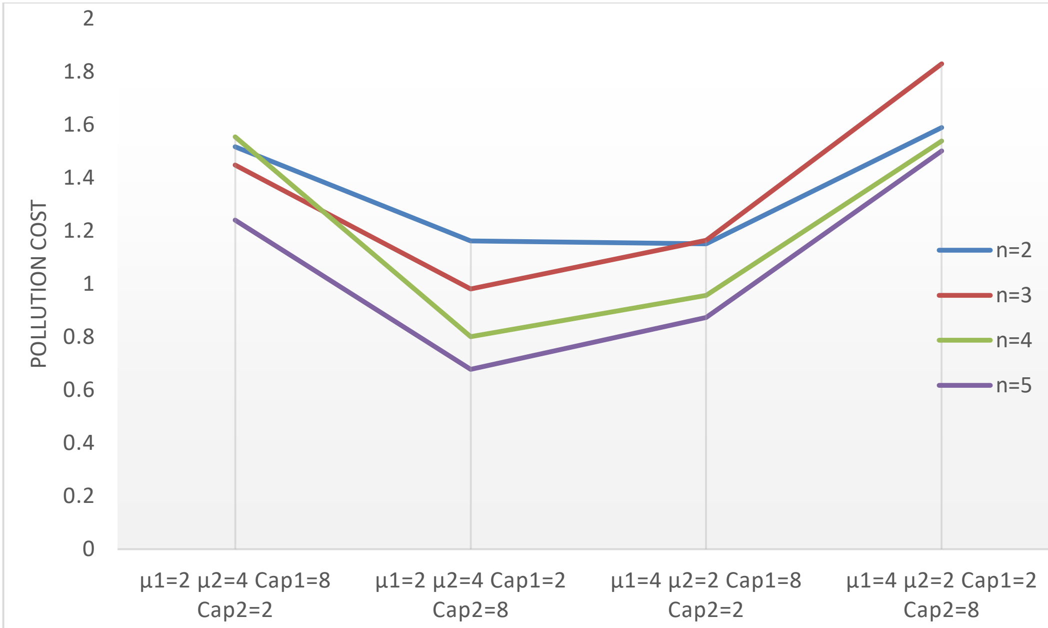



Figure 19: The behavior of the pollution cost function versus changes in the number of service providers, service speed and capacity (second scenario*)*

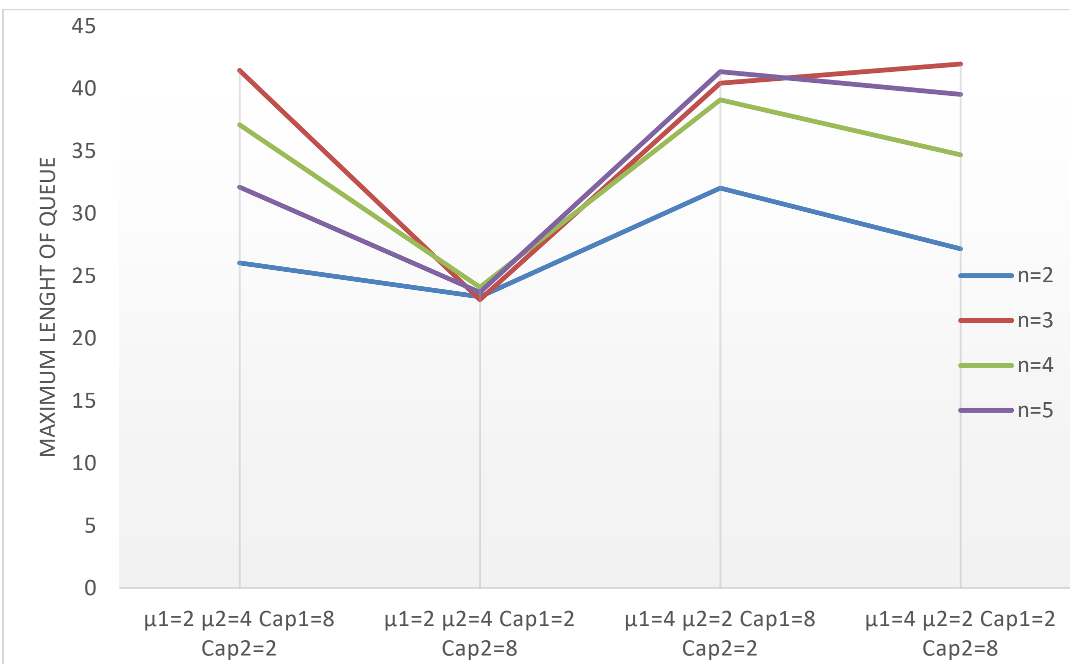



Figure 20: Maximum function of the customer's queue length versus changes in the number of service providers, service speed and capacity (second scenario)

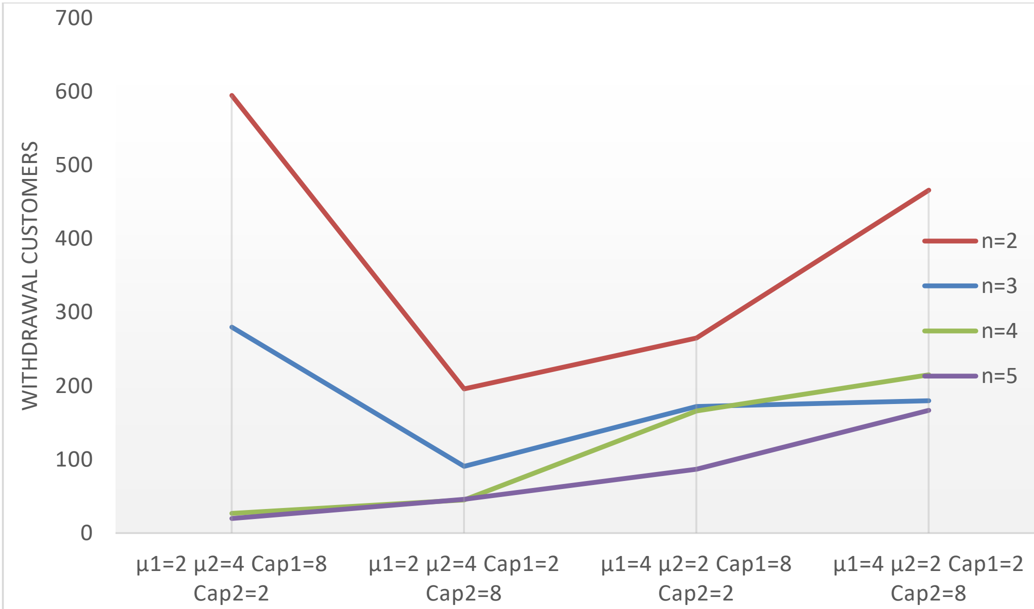



Figure 21: The number of customer withdrawal behavior versus changes in the number of service providers, service speed and capacity (second scenario)

Based on the results of Table 4 and Figures 19 to 21, we can state:

(2) Figures 19 and 20 confirm that for each level of the number of service providers, the minimum length of the queue of customers in the queue occurs when the servicemen with low capacity in the southern regions and slow servants High capacity will be located in the northern regions.

(3) As expected, as shown in Figure 21, the minimum withdrawal rate occurs when fast-serving servants are stationed at high queues.

### 6.2.3. Third scenario

As a third scenario, it is assumed that the customer entry site follows a randomized, normal distribution of two variables with $\mu = [50, 50]$ and $\sigma = [1000, 1000]$ as follows.

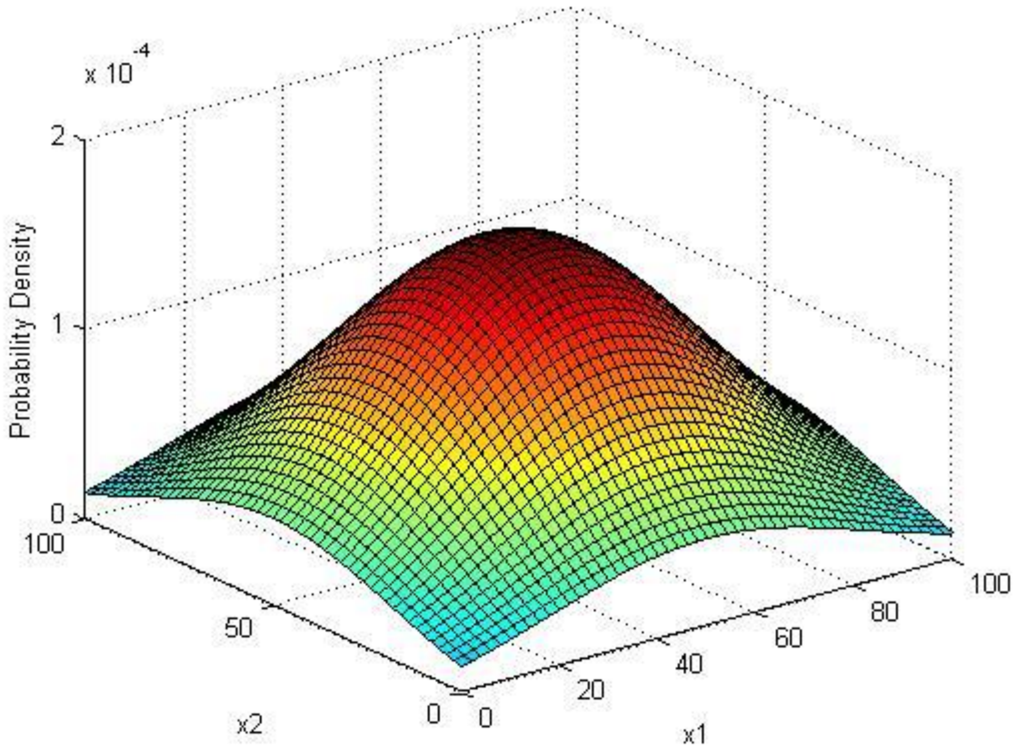



Figure 22: How to enter customers with a normal distribution of two-variable with $\mu = [50, 50]$ and $\sigma = [1000, 1000]$

Table 5: Third scenario results

| $n$ | $\mu_1$ | $\mu_2$ | $Cap_1$ | $Cap_2$ | $f_1$ | $f_2$ | $f_3$ | $CPU$ |
|---|---|---|---|---|---|---|---|---|
| 2 | 2 | 4 | 8 | 2 | 32.12336 | 1.453642 | 386 | 69.663 |
| | 2 | 4 | 2 | 8 | 32.87022 | 1.088782 | 140 | 70.589 |
| | 4 | 2 | 8 | 2 | 33.33259 | 1.131734 | 188 | 70.681 |
| | 4 | 2 | 2 | 8 | 33.20896 | 1.644486 | 400 | 70.256 |
| 3 | 2 | 4 | 8 | 2 | 41.17716 | 1.708583 | 169 | 72.189 |
| | 2 | 4 | 2 | 8 | 35.05073 | 0.953757 | 100 | 72.413 |
| | 4 | 2 | 8 | 2 | 41.52279 | 0.912542 | 65 | 72.506 |
| | 4 | 2 | 2 | 8 | 40.77797 | 1.654045 | 158 | 72.932 |
| 4 | 2 | 4 | 8 | 2 | 44.69681 | 1.338481 | 51 | 74.811 |
| | 2 | 4 | 2 | 8 | 33.99197 | 0.853892 | 46 | 74.052 |
| | 4 | 2 | 8 | 2 | 38.15916 | 0.870665 | 62 | 74.755 |
| | 4 | 2 | 2 | 8 | 42.97154 | 1.486161 | 88 | 74.893 |
| 5 | 2 | 4 | 8 | 2 | 41.1966 | 1.41198 | 58 | 74.820 |
| | 2 | 4 | 2 | 8 | 35.02931 | 0.650308 | 41 | 75.330 |
| | 4 | 2 | 8 | 2 | 37.14075 | 1.193066 | 34 | 75.174 |
| | 4 | 2 | 2 | 8 | 40.63214 | 1.389495 | 61 | 74.655 |


Figure 23: The behavior of the pollution cost function versus changes in the number of service providers, service speed and capacity (Third scenario)

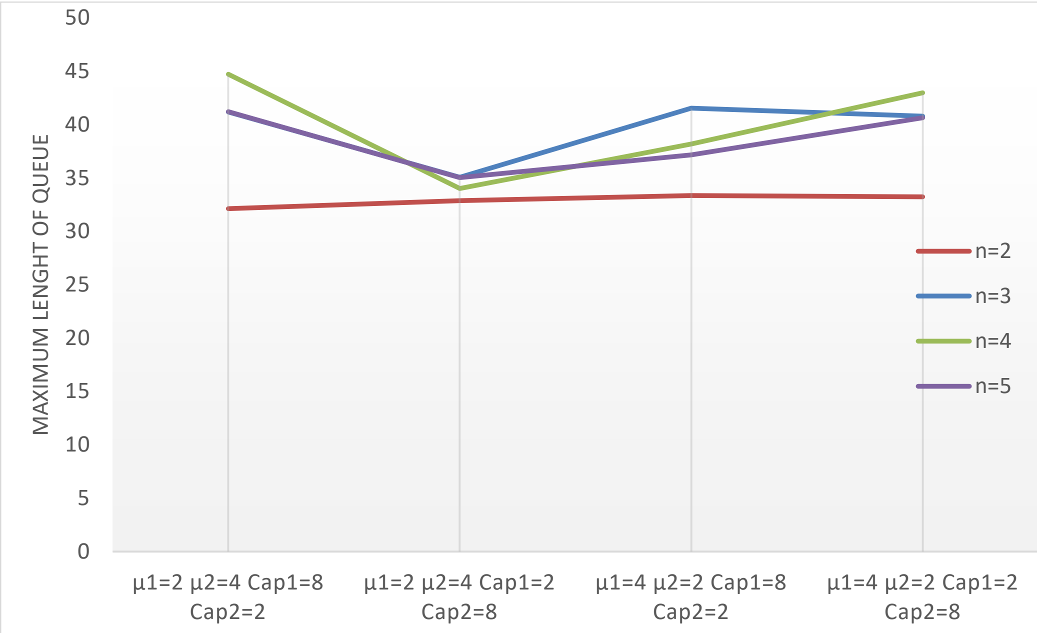



Figure 24: The function of the objective function Maximum queue length versus changes in the number of service providers, service speed and capacity (third scenario)

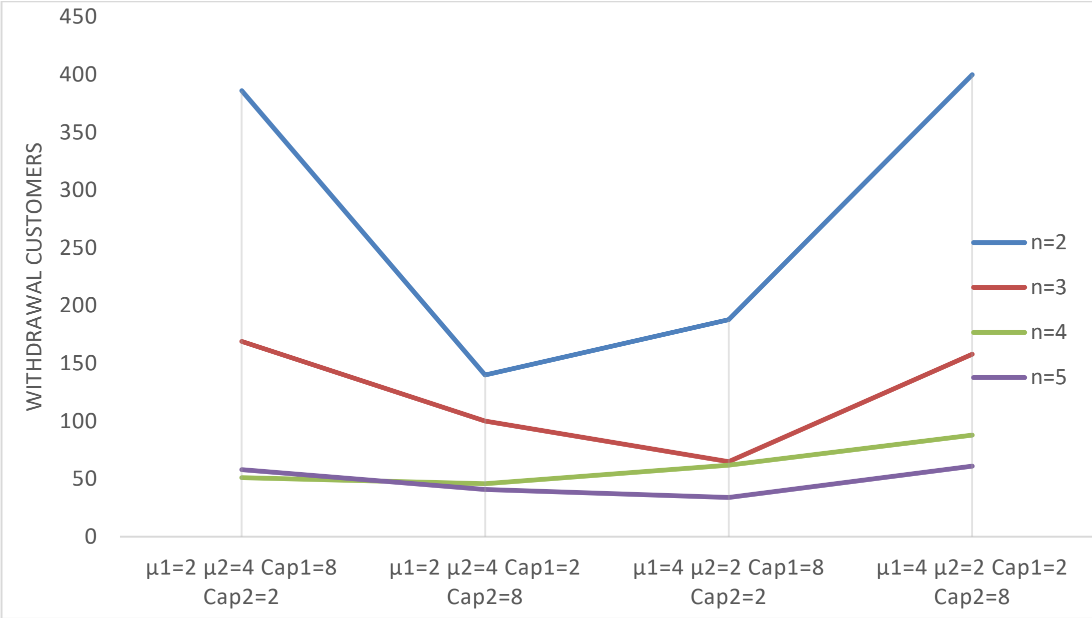



Figure 25: The number of customer withdrawal behavior versus changes in the number of service providers, service speed and capacity (third scenario)

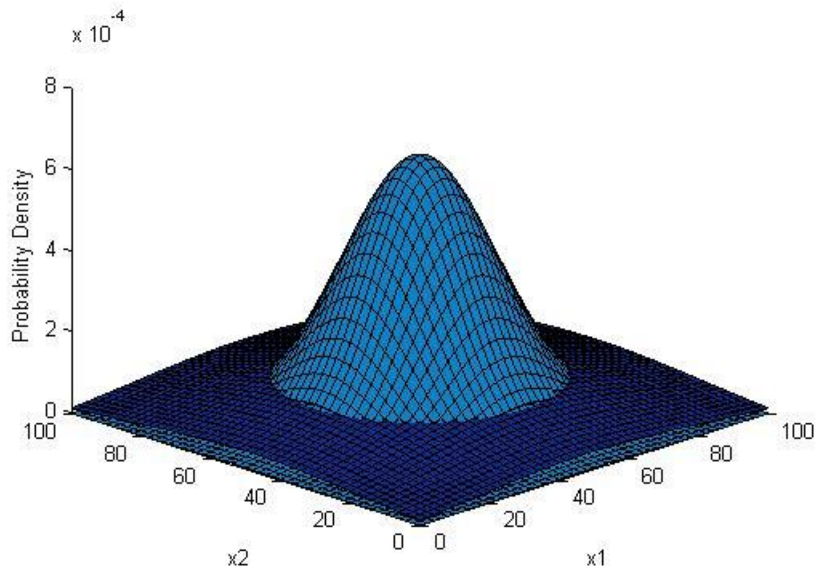
Based on Table 5 and according to Figures 23 to 25, it can be stated:

1) Considering that in the third scenario the standard deviation of the place of emergence of customers has increased, their travel time to reach the service provider is also higher than the previous two scenarios. This is especially true when the number of service providers is low, since increasing the standard deviation leads to a behavior of the normal distribution function of the two-variable to the uniform distribution function. This is illustrated in Figure 24 and in the case of two service providers, that practically changing service providers have a small impact on customer queues because the distribution of customer appearance is almost uniform.
2) Increasing the amount of dispersion the distance between the appearance of customers and the service provider will reduce the average number of customers to withdraw. Because as long as a client is more distant from reaching the service provider, there is a greater chance of shortening the queue length resulting in a decrease in withdrawal rates. This is evident in Figure 25.

### 6.2.4. Compare scenarios with each other

In this section, the results obtained from the three scenarios are compared. Figures 26 and 27 show the emergence of customer locations in the first-second scenario and the first-third scenario. Also, Figures 28 to 30 compare the variations in the maximum length of the customer's queue, the cost of pollution, and the number of customers denied in the three scenarios mentioned. Accordingly, it can be stated:

(1) Increasing dispersion in the place of customer appearance on the first objective function, namely, minimizing the maximum length of the queue, has a negative effect, so that in all 16 modes, the third scenario is more than the first and second scenarios of the customer queue length.
(2) Increasing dispersion in the place where customers are emerging has a positive impact on the number of customers. In this case, the third scenario has the lowest rate of withdrawal in most cases. This is because increasing the dispersion of the emergence of customer's increases the time it takes to reach the service provider, resulting in longer time available to reduce the queue length and thus reduce the number of customers to withdraw.




.Figure 26: Comparison of the appearance of customers in the first scenario (pale blue) and third scenario (dark blue)

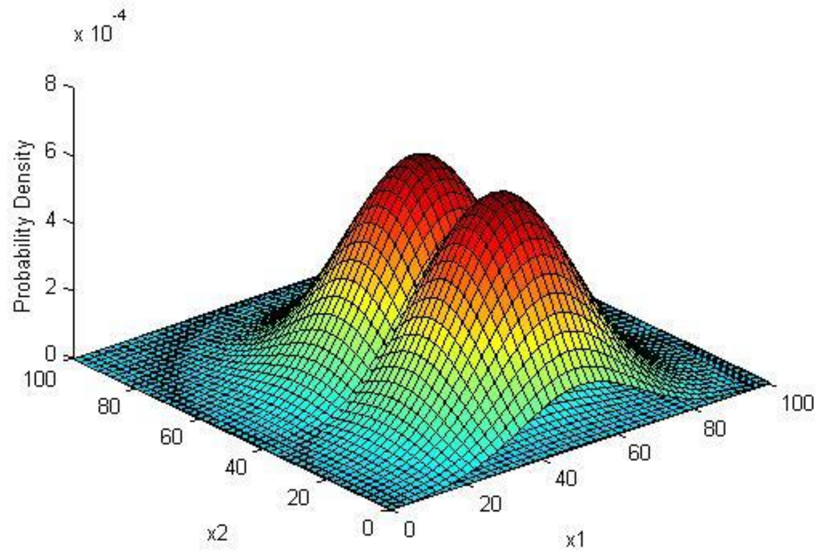



Figure 27: Comparison of the appearance of customers in the first scenario (fashion in the center) and the second .scenario (fashion alongside)

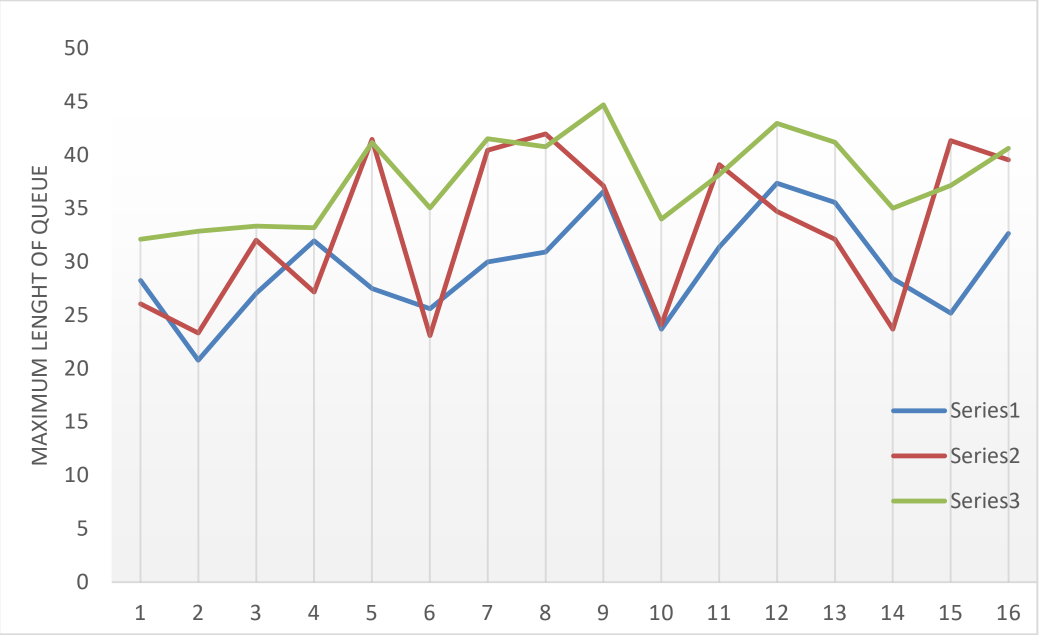



Figure 28: Comparison of Maximum Customer Length Changes for the Three Scenarios in the 16 Modes

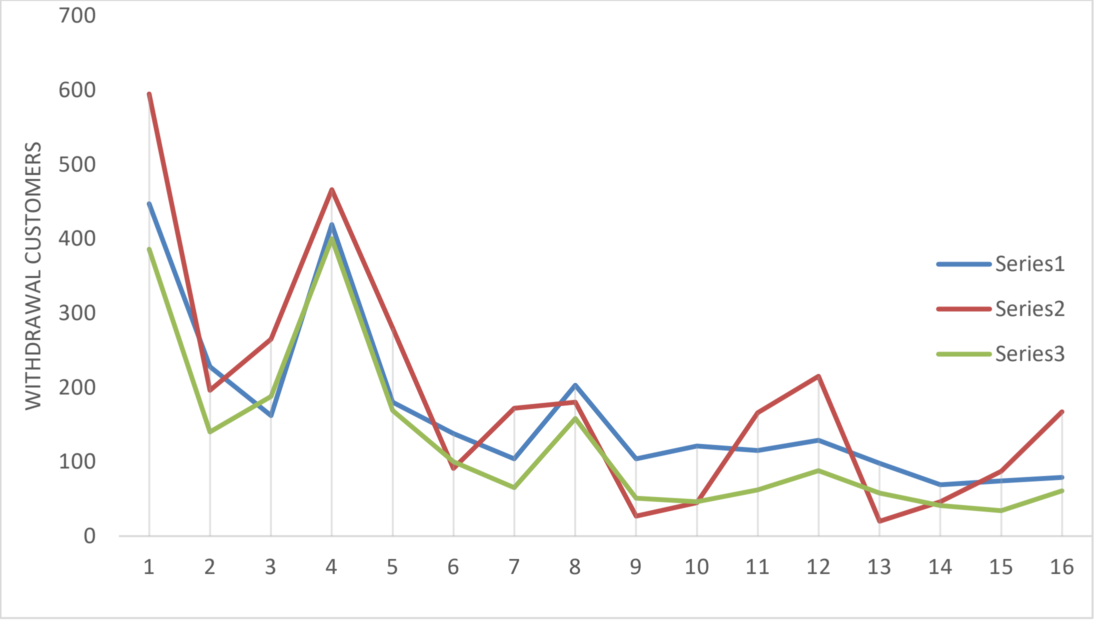



Figure 29: Comparison of changes in the number of dispatched customers for the three scenarios in the 16 states listed

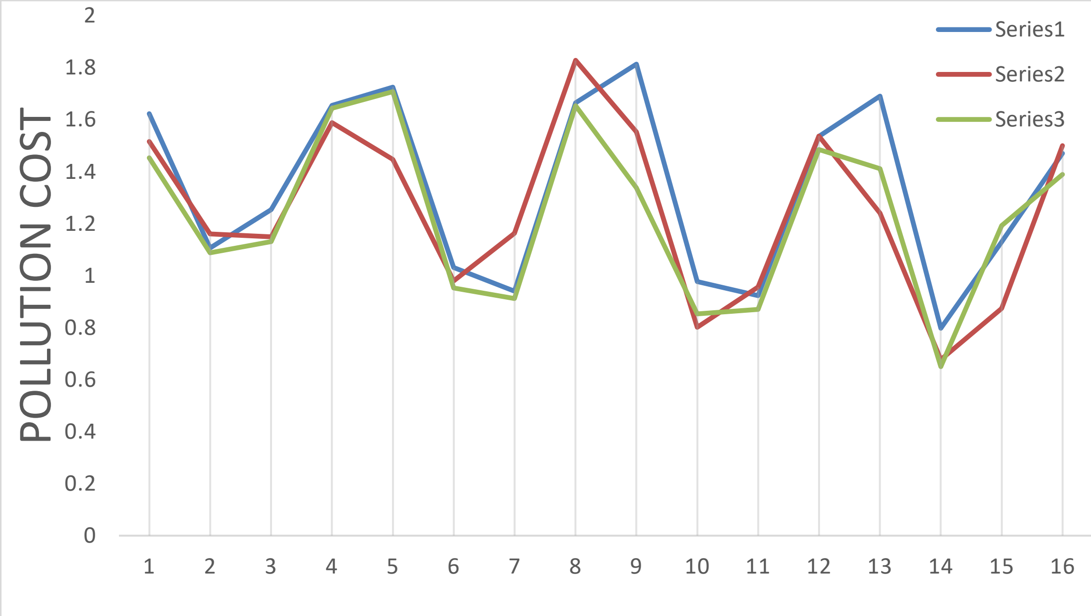



Figure 30: Comparison of pollution cost variations for the three scenarios in the 16 states listed

(3) The change in the average place of emergence of customers has a negative impact on the number of customers to withdraw. As this would dramatically increase the traffic load of service providers in a region, resulting in an increase in queuing times and, consequently, an increase in customers' rates of withdrawal.

(4) The maximum runtime among all scenarios in the second phase is 75.642 seconds. Given that the total number of possible solutions in each mode is k! / ((K-p)! P!), the complexity of the problem is with exponential growth. In addition, in each case, the process of Monte Carlo simulation is executed 100 times to achieve convergence. This makes it impossible to completely count the answers. For example, when 5 p =, 100 * 100! / (100-5)! 5! = 7528752000 simulation times are required in full search mode. But using the GA-MC hybrid approach, the optimal response was achieved at a maximum of 75.642 seconds, which indicates the high efficiency of this algorithm in optimizing the research problem.

## 7. Conclusion

In this paper, the problem of locating for hypertensive constants is examined. And in modeling the underlying question, it is assumed that each client decides to enter or dismiss before entering the facility, according to the number of people in the queue. Considering the importance of environmental issues, in addition to objectives such as minimizing the maximum queue created in the network and withdrawing customers, the goal function is to reduce the cost of transmission pollution between customers and servers. The objective functions in this model are normalized using the payoff table. The model developed in this paper has been solved in two different phases and its results have been investigated. In the first phase, using optimized genetic algorithms, optimal locations for service centers are identified on CAP and AP data and assigned to customers. In the second phase of the solution, the proposed problem presented in this study is an integer nonlinear problem (INLP) with discrete processes. In the path to optimizing such problems, there are two problems: (1) the nonlinearity of the space increases the possibility of local solutions, which reduces the possibility of achieving an optimal global response. (2) The existence of discrete random processes makes it virtually impossible to access a definite answer. To address this challenge, in this research, a hybrid optimization approach based on Monte Carlo simulation (to simulate discrete random processes) and a genetic algorithm (to avoid local optimal responses) was expressed. Finally, the proposed model was examined through numerical examples. In the same vein, numerical example assumptions were introduced including the statistical distribution of the place of emergence of customers, their statistical distribution of inputs, the classification of areas in terms of the type of service providers and the speed of service in different regions. The numerical results were presented in three different scenarios, and for each in 16 states, the sensitivity was presented and analyzed. The results are shown.

### 7.1. Future work

- In this paper, the multifunction method is used to solve the proposed nonlinear problem model. Future studies can be used to optimize the nonlinear problem of presenting alternative meta-heuristic methods.
- The exact method used for this study is a complete counting technique. The development of other precise methods, such as the Crane branch, can be considered in future studies.
- In this research, the location-allocation approach has been used, the proposed modeling approach for other location networks such as hubs can be a good topic for future studies.
- In this research, the cancellation has been considered before arrival. In many real-world systems, customer withdrawals may occur after login, and the client, after joining the queue, estimates their waiting times in line, and decides that they will remain in the queue or leave the facility. Also, after leaving the facility, the customer may generally give up receiving the service or go to a more relaxed facility. Examining these types of items can be an interesting area for future studies.

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